APPLICATION NO. 09/846,410

TITLE OF INVENTION: Multiple Data Rate Hybrid ~~Complex~~ Walsh Codes for CDMA

INVENTOR: Urbain A. von der Embse

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BACKGROUND OF THE INVENTION

This is a continuation application from No. 09/826,118 filed 01/09/2001.

I. Field of the Invention~~TECHNICAL FIELD~~

The present invention relates to CDMA (Code Division Multiple Access) for wireless cellular WAN's (wide area networks), LAN's (local area networks), PAN's (personal area networks) ~~cellular telephone and wireless data communications~~ with data rates up to multiple T1 (1.544 Mbps) and higher (>100 Mbps), and to optical CDMA with data rates in the Gbps and higher ranges. Applications are mobile, point-to-point and satellite communication networks. More specifically the present invention relates to novel multiple data rate encoders and fast decoders for Hybrid Walsh and generalized Hybrid Walsh CDMA codes. ~~algorithms for complex and hybrid complex Walsh orthogonal CDMA codes~~. ~~These algorithms generate multiple code length complex Walsh and hybrid complex Walsh orthogonal codes for use as the channelization codes for multiple data rate users.~~ These ~~new~~ algorithms and implementatioins ~~and codes~~ offer substantial improvements over the current real Walsh orthogonal variable spreading factor (OVSF) CDMA codes for the next generation wideband CDMA (W-CDMA).

II. Description of the Related Art~~CONTENTS~~

~~**BACKGROUND ART** page 1~~

~~**BACKGROUND ART**~~

Current art is represented by the work on orthogonal spreading factor (OVSF) real Walsh codes for 3G CDMA2000 and W-CDMA, ~~wideband CDMA (W-CDMA) for the third generation CDMA (G3)~~ proposed standards for the fourth generation CDMA (G4) ~~candidates and for broadband wireless communications, and the previous work on the real Walsh fast transform algorithms. These are~~ documented in ~~references 1,2,3,4,5. Reference 1 is an issue of~~ listed IEEE journals including the IEEE Journal on selected areas in communications August 2000 Vol. 18 No. 8 "Wideband CDMA" ~~communications journal~~ devoted to wideband CDMA including OVSF, and ~~.~~ the listed patents. ~~References 2 and 3 are issues of the IEEE communications magazine that are devoted to "Multiple Access for Broadband Networks" and "Wideband CDMA". Reference 4 is an issue of the IEEE personal communications devoted to "Third Generation Mobile Systems in Europe". Reference 5 is the widely used reference on real Walsh technology which includes algorithms for the fast Walsh transform. The new complex Walsh and hybrid complex Walsh orthogonal CDMA codes being addressed in this invention for application to multiple data rate users, have been disclosed in a previous patent application [6] for constant data rate communications.~~

Current art ~~using~~ uses real Walsh orthogonal CDMA channelization codes to generate OVSF codes for multiple data rate users and ~~is represented by the scenario described in the~~

~~following with the aid of equations (1) and (2) and FIG. 1,2,3,4. This scenario~~ considers CDMA communications spread over a common frequency band for each of the communication channels. With OVSF ~~These~~ the CDMA communications channels for each of the multiple rate users are defined by assigning a unique real Walsh orthogonal spreading code to each user. This real Walsh code has a maximum length of N chips with $N=2^M$ where M is an integer, with shorter lengths of 2,4,…,N/2 for the higher data rate users. These multiple length real Walsh codes have limited orthogonality properties and occupy the same frequency band. These Walsh encoded user signals are summed and then re-spread over the same frequency band by pseudo-noise (PN) codes, to generate the CDMA communications signal which is modulated and transmitted. The communications link consists of a transmitter, propagation path, and receiver, as well as interfaces and control.

~~It is assumed that the communication link is in the communications mode with the users communicating at symbol rates equal to the code repetition rates of their respective communications channels and that the synchronization is sufficiently accurate and robust to support this communications mode. In addition, the power differences between users due to differences in data rates and in communication link budget parameters is assumed to be incorporated in the data symbol amplitudes prior to the CDMA encoding in the CDMA transmitter, and the power is uniformly spread over the wideband by proper selection of the CDMA pulse waveform. It is self evident to anyone skilled in the CDMA communications art that these communications mode assumptions are both reasonable and representative of the current CDMA art and do not limit the applicability of this invention.~~

Transmitter equations **(1)** describe a representative real Walsh CDMA encoding for multiple data rate users for the transmitters in FIG. **1A,1B,1C,2A**. Multiple code length real

Walsh codes are defined in **1** in equations **(1)**. The multiple data rate menu in **2** lists the user group m=0,1,2,. . .,M-1, data symbol rate R_s, code length, and the number of symbols transmitted over each N-chip reference code length. In this invention disclosure it is assumed the user data symbols have the same symbol data encoding which means the multiple data rate users can be categorized according to their symbol rate.

~~These equations represent a considerably more sophisticated and improved implementation of current OVSF CDMA communications which has been developed to help support the new invention for complex Walsh and hybrid complex Walsh CDMA orthogonal codes.~~

~~Lowest data rate users are assumed to communicate at the lowest symbol rate equal to the code repetition rate of the N chip real Walsh code, which means they are assigned N chip code vectors from the NxN real Walsh code matrix W_N in **1** for their channelization codes. Higher data rate users will use shorter real Walsh codes. The reference real Walsh code matrix W_N has N Walsh row code vectors $W_N(c)$ each of length N chips and indexed by c=0,1,…,N-1, with $W_N(c)=[W_N(c,1),…,W_N(c,N)]$ wherein $W_N(c,n)$ is chip n of code u. Walsh code chip n of code vector u has the possible values $W_N(c,n)=+/-1$.~~

~~Multiple data rate menu in **2** lists the possible user data symbol rates R_s and the corresponding code lengths and symbols transmitted over each N chip reference code length. User symbol rate $R_s=1/NT$ is the code repetition rate 1/NT of the N chip code over the code time interval NT. User data rate R_b in bits/second is equal to $R_b=R_sb_s$ where b_s is the number of data bits encoded in each data symbol. Assuming a constant b_s for all of the multiple data rate users, the user data rate becomes directly proportional to the user symbol rate $R_b\sim R_s$ which means the user symbol rate menu in 1 is equivalent to the user data rate menu.~~

User data symbols and channelization codes are listed in **3**. for the multiple data rate users. ~~Users are grouped into the data rate categories corresponding to their respective code chip~~

~~lengths 2,4,8,…,N/2,N chips. User groups are indexed by m=1,2,…,M where group m consists of all users with $N(m)=2^m$ chip length codes drawn from the N(m)xN(m) real Walsh code matrix $W_{N(m)}$.~~ Users u_m ~~with~~in group m have their index u_m ~~are identified by the index u_m which is~~ set equal to the Walsh channelization code vector index in $W_{N(m)}$. Code chip n_m of the user code u_m is equal to $W_{N(m)}(u_m, n_m)$ where $n_m=0,1,2,\ldots,N(m)-1$ is the chip index. User data symbols $Z(u_{m,k_m})$ are indexed by u_{m,k_m} where the index $k_m=0,1,2,\ldots,N/N(m)-1$ identifies the data symbols of u_m which are transmitted over the N chip code block. The total number of user data symbols transmitted per N chip block is N. ~~which means the number of channel assignments $\{u_m, m=1,2,\ldots,M\}$ will be less than N for multiple data rate CDMA communications when there is at least one user using a higher data rate.~~

Current multiple data rate real Walsh CDMA encoding **(1)**
~~for transmitter~~

1 N chip Walsh code block

W_N = Walsh NxN orthogonal code matrix consisting of N rows of N chip code vectors
= [$W_N(c)$] matrix of row vectors $W_N(c)$
= [$W_N(c,n)$] matrix of elements $W_N(c,n)$

$W_N(c)$ = Walsh code vector c for $c=0,1,\ldots,N-1$
= [$W_N(c,0)$, $W_N(c,1)$, …, $W_N(c,N-1)$]
= 1xN row vector of chips $W_N(c,0),\ldots,W_N(c,N-1)$

$W_N(c,n)$ = Walsh code c chip n
= ~~+/+~~+/- 1 possible values

2 Multiple data rate menu

~~N-chip real Walsh symbol rate~~

~~R_s = User symbol rate, symbols/second~~

~~= 1/NT where T = Chip repetition interval~~

~~Symbol rate menu for multiple data rates~~

Group m	Symbol rate R_s, Symbols/second	Code length, chips	Symbols per N chips
~~R_s~~ 0 =	1/2T	2	N/2
1 =	1/4T	4	N/4
2 =	1/8T	8	N/8
	⋮	⋮	⋮
M-2 =	~~1~~2/~~2~~NT	N/2	2
M-1 =	1/NT	N	1

where 1/T = chip rate of real Walsh code

T = chip repetition interval

3 User data symbols and channelization codes

Users are categorized into M groups according to the number of code chips.

m = Index of the user groups

= ~~1~~0, ~~2~~1, …., M-1

u_m = One of ~~up to~~ $N(m)=2^m$ possible users in group m

N(m) = Number of code chips ~~for the codes in the~~ users in group m

= Number of users allowed in group m

= $2^{\cancel{m+1}}$

User data symbols

$Z(u_{m,k_m})$ = User u_m data symbol k_m

k_m = Index for the user data symbols over the N chip code block, for a user from group m

= 0,1,2,...., N/N(m)-1

User channelization codes within each group are selected from a subset of the orthogonal codes in the Walsh code matrix.

$W_{N(m)}(u_m)$ = Walsh $1x2^m$ dimensional code vector u_m in the N(m)xN(m) Walsh code matrix, for user u_m in the group m

$W_{N(m)}(u_m, n_m)$ = User u_m code chip n_m=0,1,2,....,N(m)-1

4 Real Walsh encoding and channel combining

$\tilde{Z}(n)$ = Real Walsh CDMA encoded chip n

$$= \sum_{m=1}^{M} \sum_{u_m} Z(u_{m,k}) \; W_{N(m)}(u_m, \; n=n_m+ \; k_m \; N(m) \;)$$

5 PN scrambling

$P_2(m)$ = long PN real code

$P_R(n)$, $P_I(n)$ = ~~PN~~ short PN complex code chip n ~~for~~ real, imaginary components~~axes~~

~~Z(n) = PN scrambled real Walsh encoded data chips after summing over the users~~

$$\cancel{= \sum_u \tilde{Z}(n) P_2(n)[P_R(n) + j P_I(n)]}$$

$$\cancel{= \sum_u \tilde{Z}(n) \, sign\{P_2(n)\} [sign\{P_R(n)\} + j \, sign\{P_I(n)\}]}$$

$Z_n(n)$ = $\tilde{Z}(n)$ $P_2(n)$ $[P_R(n)+j \; P_I(n)]$

$$= \tilde{Z}(n)\ \mathrm{sign}\{P_2(n)\}[\mathrm{sign}\{P_R(n)\} + j\ \mathrm{sign}\{P_I(n)\}]$$

= Real Walsh CDMA encoded complex chips after PN scrambling

where $j = \sqrt{-1}$

Walsh encoding and channel combining in **4** encodes each of the users $\{u_m\}$ and their data symbols $\{Z(u_{m,k_m})\}$ with a Walsh code $W_{N(m)}(u_m)$ drawn from the group m of the N(m) chip channelization codes where u_m is the user code. A time delay of $k_m N(m)$ chips before start of the real Walsh encoding of the data symbol k_m in each of the user channels, is required for implementation of the multiple data rate user real Walsh encoding and for the summation of the encoded data chips over the users. Output of this multiple data rate real Walsh encoding and summation over the multiple data rate users is the set of real Walsh CDMA encoded chips $\{\cancel{\tilde{Z}}\tilde{Z}_n\cancel{(}n)\}$ over the N chip block.

PN scrambling of the real Walsh CDMA encoded chips in **5** is accomplished by encoding the $\{\cancel{\tilde{Z}}\tilde{Z}_n(n)\}$ with a long code real PN and a short code complex PN. ~~which is constructed as the complex code sequence [$P_R(n)+jP_I(n)$] wherein $P_R(n)$ and $P_I(n)$ are independent PN sequences used for the real and imaginary axes of the complex PN. These PN codes are 2-phase with each chip equal to +/-1 which means PN encoding consists of sign changes with each sign change corresponding to the sign of the PN chip. Encoding with PN means each chip of the summed Walsh encoded data symbols has a sign change when the corresponding PN chip is -1, and remains unchanged for +1 values. This operation is described by a multiplication of each chip of the summed Walsh encoded data symbols with the sign of the PN chip. Purpose of the PN encoding for complex data symbols is to provide scrambling of the summed Walsh encoded data symbols as well as isolation~~

~~between groups of users.~~ Output of this real Walsh CDMA encoding followed by the complex PN scrambling are the CDMA encoded chips over the N chip block {Z(n)}.

Receiver equations **(2)** describe a representative multiple data rate real Walsh CDMA decoding for the receiver in FIG. **3A,3B,4A**. ~~The receiver front end **5** provides estimates $\{\hat{Z}_n(n)\}$ of the transmitted real Walsh CDMA encoded chips {Z(n)}. Orthogonality property~~ In **6** the multiple rate codes are orthogonal with respect to the user codes within a group and also between code groups for all code repetitions over the N chip code block. The PN codes **7** have the useful decoding property that the square of each real code chip is unity which is used in the decoding algorithms **8** that perform the inverse of the signal processing for the encoding in equations **(1)** to recover estimates $\{\hat{Z}(u_{m,k_m})\}$ of the transmitter user symbols $\{Z(\mathbf{u}_{m,k_m})\}$ from the received estimates $\hat{Z}(n)$ of the transmitted real Walsh CDMA encoded chips Z(n). ~~is expressed as a matrix product of the real Walsh code chips or equivalently as a matrix product of the Walsh code chip numerical signs, for any of the 2,4,8,…,N/2,N chip real Walsh channelization codes and their repetitions over the N chip code block. These codes are orthogonal with respect to the user codes within a group. They are also orthogonal between code groups for the allowable subsets of code assignments to the users, for all code repetitions over the N chip code block. This means that the allowable codes~~ $\cancel{\{u_m\}}$ ~~in group m are orthogonal to the allowable codes~~ $\cancel{\{u_{m+p}\}}$ ~~in group m+p for all code repetitions of the codes~~ $\cancel{\{u_m\}}$ ~~over the N chip code block, for p≥0.~~

~~The 2-phase PN codes **7** have the useful decoding property that the square of each code chip is unity which is equivalent to observing that the square of each code chip numerical sign is~~

~~unity. Decoding algorithms 8 perform the inverse of the signal processing for the encoding in equations (1) to recover estimates {$\hat{Z}(u_{m,k_m})$} of the transmitter user symbols {$Z(u_{m,k_m})$}.~~

Current multiple data rate real Walsh CDMA decoding ~~for receiver~~ **(2)**

5 Receiver front end provides estimates ~~{$\hat{\mathbf{Z}}_n(\mathbf{n}) = \hat{\mathbf{R}}_n(\mathbf{n}) + j\hat{\mathbf{I}}_n(\mathbf{n})$}~~ {$\hat{Z}(n)$} of the encoded transmitter chip symbols {Z(n)}

6 Orthogonality properties of the set of real Walsh {2x2, 4x4, 8x8, ...NxN} matrices

The N(m)xN(m) Walsh code matrices ~~for all m~~ are orthogonal

$$N(m)^{-1}\sum_{n_m} W_{N(m)}(\hat{c}_m, n_m)\, W_{N(m)}(n_m, c_m) = \delta(\hat{c}_m, c_m)$$

where $c_m, n_m = 0,1,\ldots,N(m)$

$\delta(\hat{c}_m, c_m)$ = Delta function of $\hat{c}_m$ and c_m

= 1,0 for $\hat{c}_m = c_m$, otherwise

The N(m)xN(m) and N(m+p)xN(m+p) Walsh code matrices for all m and p≥0 are orthogonal for a subset of codes {u_m} and {u_{m+p}}

$$N(m)^{-1}\sum_{n_m} W_{N(m)}(u_m, n_m) \bullet W_{N(m+p)}(u_{m+p},\ n_{m+p} = n_m + k_m N(m))$$

$$= 0 \text{ for } k_m = 0,1,2\ldots, N/N(m)-1$$

7 PN decoding property for $P(n)=P_2(n), P_R(n), P_I(n)$

$P(n)P(n) = \mathrm{sign}\{P(n)\ \mathrm{sign}\{P(n)\}$

$= 1$

8 Decoding algorithm

$$\hat{Z}(u_{m,k_m}) =$$

$$N^{-1}\sum_{n_m}\hat{Z}_n(n)\,\mathrm{sign}\{P_2\,(n)\}[\mathrm{sign}\{P_R(n)\}-j\,\mathrm{sign}\{P_I(n)\}]\bullet$$

$$\mathrm{sign}\{W_{N(m)}(\;n=n_m+k_mN(m),\;u_m\;)\}$$

= Receiver estimate of the transmitted complex data symbol $Z(u_{m,k_m})$

FIG. **1A** CDMA transmitter block diagram is representative of a current CDMA transmitter ~~which includes an implementation of the current~~ for multiple data rate real Walsh CDMA channelization encoding in equations **(1)**. This block diagram becomes a representative implementation of the CDMA transmitter ~~which implements~~ for the ~~new~~ multiple data rate ~~complex~~ Hybrid Walsh and generalized ~~h~~Hybrid ~~complex~~ Walsh CDMA encoding, when the current multiple data rate real Walsh CDMA encoding **13** is replaced by the ~~new~~ multiple data rate ~~complex~~ Hybrid Walsh and generalized ~~h~~Hybrid ~~complex~~ Walsh CDMA encoding ~~of this invention.~~

Signal processing starts with the stream of user input data words **9**. Frame processor **10** accepts these data words and performs the encoding and frame formatting, and passes the outputs to the symbol encoder **11** which encodes the frame symbols into amplitude and phase coded symbols $\{Z(u_{m,k})\}$ **12**. These symbols **12** are the inputs to the current multiple data rate real Walsh CDMA encoding in equations (1). Inputs $\{Z(u_{m,k})\}$ **12** are real Walsh encoded, summed over the users, and scrambled by complex PN in the current multiple date rate real Walsh CDMA encoder **13** to generate the complex output chips $\{Z(n)\}$ **14**. This encoding **13** is a representative implementation of equations (1). These output chips $Z(n)$ are waveform modulated **15** to generate the

analog complex signal z(t) which is single sideband upconverted, amplified, and transmitted (Tx) by the analog front end of the transmitter **15** as the real waveform v(t) **16** at the carrier frequency f_0 whose amplitude is the real part of the complex envelope of the baseband waveform z(t) multiplied by the carrier frequency and the phase angle ϕ accounts for the phase change from the baseband signal to the transmitted signal.

FIG. **1B** is a representative wireless cellular communication network application of the generalized CDMA transmitter in FIG. **1A**. FIG. **1B** is a schematic layout of part of a CDMA network which depicts cells **101,102,103,104** that partition this portion of the area coverage of the network, depicts one of the users **105** located within a cell with forward and reverse communications links **106** with the cell-site base station **107,** depicts the base station communication links **108** with the MSC/WSC **109,** and depicts the MSC/WSC communication links with another base station **117,** with another MSC/WSC **116,** and with external elements **110,111,112,113,114,115.** One or more base stations are assigned to each cell or multiple cells or sectors of cells depending on the application. One of the base stations **109** in the network serves as the MSC (mobile switching center) or WSC (wireless switching center) which is the network system controller and switching and routing center that controls all of user timing, synchronization, and traffic in the network and with all external interfaces including other MSC's. External interfaces could include satellite **110,** PSTN (public switched telephone network) **111,** LAN (local area network) **112,** PAN (personal area network) **113,** UWB (ultra-wideband network) **114,** and optical networks **115.** As illustrated in the figure, base station **107** is the nominal cell-site station for cells i-2, i-1, i, i+1 identified as **101,102,102,104,** which means it is intended to service these cells with overlapping coverage from other base stations. The cell topology and coverage depicted in the figure are intended

to be illustrative and the actual cells could be overlapping and of differing shapes. Cells can be sub-divided into sectors. Not shown are possible subdivision of the cells into sectors and/or combining the cells into sectors. Each user in a cell or sector communicates with a base station which should be the one with the strongest signal and with available capacity. When mobile users cross over to other cells and/or are near the cell boundary a soft handover scheme is employed for CDMA in which a new cell-site base station is assigned to the user while the old cell-site base station continues to service the user for as long as required by the signal strength.

Fig. **1C** depicts a representative embodiment of the CDMA transmitter signal processing in **13,15** of FIG. **1A** for the forward and reverse CDMA links **106** in FIG. **1B** between the base station and the users for CDMA2000 and W-CDMA that implements the CDMA coding for synchronization, real Walsh channelization, and scrambling of the data for transmission. Depicted are the principal signal processing from **13,15** in FIG. **1A** that is relevant to this invention disclosure. CDMA2000 and W-CDMA use real Walsh codes **120** for channelization of the data expressed in an OVSF layered format.

FIG. **1C** data inputs **12** in FIG. **1A** to the transmitter CDMA signal processing are the inphase data symbols R **118** and quadrature data symbols I **119** of the complex data symbols Z(u) from the block interleaving processing in the transmitter in **12** in FIG. **1A.** As described previously in equation **(1)** in greater detail, a real Walsh code **120** ranging in length from N=4 to N=512 chips spreads and channelizes the data by encoding **121** the inphase and quadrature data symbols with rate R=N codes corresponding to the channel assignments of the data chips. A long PN code **122** encodes the inphase and quadrature real Walsh encoded chips **123.** The long PN code **122** is a PN code sequence intended to provide separation of the cells and sectors and to

provide protection against multipath. Long PN codes **122** for IST-95/IST-95A use code segments from a 42 bit maximal-length shift register code with code length (2^42-1). The separation between code segments is sufficient to make them statistically independent. These codes can be converted to complex codes by using the code for the real axis and a delayed version of the code for the quadrature axis whereupon the encoding **123** is replaced by a complex multiply operation similar to the short code complex multiply **126** and in **4** in Equation **(1)**. Different code segments are assigned to different cells or sectors to provide statistical independence between the communications links in different cells or sectors. This long PN code covering of the real Walsh encoded chips is followed by a short complex PN code covering in **124,125,126**. Short PN codes are used for scrambling and synchronization of CDMA code chips from the real Walsh encoding of the data symbols after they are multiplied by a long code. These codes include real and complex valued segments of maximal-length shift register sequences and segments of complex Gold codes which range in length from 256 to 38,400 chips and also are used for user separation and sector separation within a cell. Short PN codes also include Kasami sequen**ces,** Kerdock codes, and Golay sequences. This complex PN short code encodes the inphase and quadrature chips with a complex multiply operation **126** as described in **4** in Equation **(1)**. Outputs are inphase and quadrature components of the complex chips which have been rate R=1 phase coded with both the long and short PN codes. Low pass filtering (LPF), summation (Σ) over the Walsh channels for each chip symbol, modulation of the chip symbols to generate a digital waveform, and digital-to-analog (D/A) conversion operations **127** are performed on these encoded inphase and quadrature chip symbols to generate the analog inphase x(t) signal **128** and the quadrature y(t) signal **129** which are the components of the complex signal z(t)=x(t)+jy(t) where j=√-1. In equations **(1)** the code summation is equivalently performed by

the real Walsh encoding. This complex signal z(t) is single-sideband up-converted to an IF frequency and then up-converted by the RF frequency front end to the RF signal v(t) **133** which is defined in **16** in FIG. **1A**. Single sideband up-conversion of the baseband signal is performed by multiplication of the inphase signal x(t) with the cosine of the carrier frequency f_0 **130** and the quadrature signal y(t) by the sine of the carrier frequency **131** which is a 90 degree phase shifted version of the carrier frequency, and summing **132** to generate the real signal v(t) **133.**

FIG. **1C** depicts an embodiment of the current CDMA transmitter art and with current art signal processing changes this figure is representative of other current art CDMA transmitter embodiments for this invention disclosure. Other embodiments of the CDMA transmitter include changes in the ordering of the signal processing, single channel versus multi-channel real Walsh encoding, summation or combining of the Walsh channels by summation over like chip symbols, analog versus digital signal representation, baseband versus IF frequency CDMA processing, the order and placement in the signal processing thread of the Σ, LPF, and D/A signal processing operations, and the up-conversion processing. The order of the rate R=1 PN multiplies in FIG. **1C** can be changed since the covering operations implemented by the multiplies are linear in phase, which means the short PN code complex multiply **124,125,126** in FIG. **1C** can occur prior to the long PN code multiply **122,123** and moreover the long PN code can be complex with the real multiply **123** replaced by the equivalent complex multiply **126.**

It should be obvious to anyone skilled in the communications art that this example implementation in FIG. **1A,1B,1C** clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that

this example is representative of the other possible signal processing approaches~~.~~.

FIG. **2A** multiple data rate real Walsh CDMA encoding is a representative implementation ~~of~~ algorithm for the multiple data rate real Walsh CDMA encoding **13** in FIG. **1A, 120,121** in FIG. **1C,** and in equations **(1)**. Inputs are the complex user data symbols {Z(u)} **17**. Encoding of each user by the corresponding Walsh code is described in **18** by the implementation of transferring the sign of each Walsh code chip to the user data symbol followed by a 1-to-N expander $1\uparrow N$ of each data symbol into an N chip sequence using the sign transfer of the Walsh chips. The sign-expander operation **18** generates the N-chip sequence $Z(u_{m,k_m})\,\mathrm{sgn}\{W(u_m, (n=n_m+k_m N(m))\}$ for $n=0,1,\ldots,N-1$ for each user $\{u_m\}$. This Walsh encoding serves to spread each user data symbol into an orthogonally encoded chip sequence which is spread over the CDMA communications frequency band. The Walsh encoded chip sequences for each of the user data symbols are summed over the users **19** followed by PN encoding with the ~~scrambling~~ sequence $P_2(n)\,[P_R(n)+jP_I(n)]$ **20**. ~~PN encoding is implemented by transferring the sign of each PN chip to the summed chip of the Walsh encoded data symbols.~~ Output is the stream of complex multiple data rate real Walsh CDMA encoded chips {Z(n)} **21**.

It should be obvious to anyone skilled in the communications art that this example implementation in FIG. **2** clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

FIG. **3A** CDMA receiver block diagram is representative of a current CDMA receiver which includes an implementation of the current multiple data rate real Walsh CDMA decoding in equations

(2). This block diagram becomes a representative implementation of the CDMA receiver which implements the new multiple data rate complex Walsh and hybrid complex Walsh CDMA decoding when the current multiple data rate real Walsh CDMA decoding **27** is replaced by the new multiple data rate complex Walsh and hybrid complex Walsh CDMA decoding of this invention.

FIG. **3A** signal processing starts with the user transmitted wavefronts incident at the receiver (Rx) antenna **22** for the users $\{u_m\}$. These wavefronts are combined by addition in the antenna to form the receive (Rx) signal $\hat{v}(t)$ at the antenna output **22** where $\hat{v}(t)$ is an estimate of the transmitted signal v(t) **16** in FIG. 1, that is received with errors in time Δt, frequency Δf, phase $\Delta\theta$, and with an estimate $\hat{z}(t)$ of the transmitted complex baseband signal z(t) **16** in FIG. 1. This received signal $\hat{v}(t)$ is amplified and downconverted by the analog front end **23** and then synchronized and analog-to-digital (ADC) converted **24**. Outputs from the ADC are filtered and chip detected **25** by the fullband chip detector, to recover estimates $\{\hat{Z}(n)\}$ **26** of the transmitted signal which is the stream of complex CDMA encoded chips {Z(n) **14** in FIG. 1. CDMA decoder **27** implements the algorithms in equations (2) by stripping off the PN code(s) and decoding the received CDMA real Walsh orthogonally encoded chips to recover estimates $\{\hat{Z}(u_{m,k_m})\}$ **28** of the transmitted user data symbols $\{Z(u_{m,k_m})\}$ **12** in FIG. 1. These estimates **28** are processed by the symbol decoder **29** and the frame processor **30** to recover estimates **31** of the transmitted user data words.

Fig. **3B** depicts a representative embodiment of the receiver signal processing **27** in FIG. **3A** for the forward and reverse CDMA links **106** in FIG. **1B** between the base station and the user for

CDMA2000 and W-CDMA that implements the CDMA decoding for the long and short codes, the real Walsh codes, and for recovering estimates $\hat{R}, \hat{I}$ **148,149** of the transmitted inphase and quadrature data symbols R **118** and I **119** in FIG. **1C**. Depicted are the principal signal processing that is relevant to this invention disclosure. Signal input $\hat{v}(t)$ **134** in FIG. **3B** is the received transmitted CDMA signal v(t) **16** in FIG. **1A** and **133** in FIG. **1C**. The signal is handed over to the inphase mixer which multiplies $\hat{v}(t)$ by the cosine **135** of the carrier frequency f_0 followed by a low pass filtering (LPF) **137** which removes the mixing harmonics, and to the quadrature mixer which multiplies $\hat{v}(t)$ by the sine **136** of the carrier frequency f_0 followed by the LPF **137** to remove the mixing harmonics. These inphase and quadrature mixers followed by the LPF perform a Hilbert transform on v(t) to down-convert the signal at frequency f_0 and to recover estimates $\hat{x}, \hat{y}$ of the inphase component x(t) and the quadrature component y(t) of the transmitted complex baseband CDMA signal z(t)=x(t)+jy(t) in **128,129** FIG. **1C** The $\hat{x}(t)$ and $\hat{y}(t)$ baseband signals are analog-to-digital (D/A) **140** converted and demodulated (demod.) to recover the transmitted inphase and quadrature baseband chip symbols. The complex short PN code cover is removed by a complex multiply **143** with the complex conjugate of the short PN code implemented by using the inphase short code **141** and the negative of the quadrature short code **142** in the complex multiply operation **143**. The long PN code cover is removed by a real multiply **145** with the long code **144** implemented as (+/-) sign changes to the chip symbols since this is a binary 0,1 code. The decovered chip symbols are rate R=1/N decoded by the real Walsh decoders **146** using the real Walsh code **147** which implement the real Walsh decoding **36** in FIG. **4**. Decoded output symbols are the estimates $\hat{R}, \hat{I}$ **148,149** of the inphase data symbols R and

the quadrature data symbols I from the transmitters **12** FIG. **1A** and **118,119** FIG. **1C**.

FIG. **3B** depicts an embodiment of the current CDMA receiver art and with current art signal processing changes this figure is representative of other current art CDMA receiver embodiments for this invention disclosure. Other embodiments of the CDMA receiver include changes in the ordering of the signal processing, analog versus digital signal representation, down-conversion processing, baseband versus IF frequenncy CDMA processing, order and placement in the signal processing thread of the Σ, LPF, and A/D signal processing operations, and single channel versus multi-channel real Walsh decoding. Code decovering is implemented as rate R=1 code multiply operations which implement the phase subtraction of the code symbols from the chip symbols. The order of the rate R=N code multiplies in FIG. **3** can be changed since the covering operations implemented by the multiplies are linear in phase, which means the short code complex multiply **141,142,143** in FIG. **3B** can occur prior to the long code multiply **144,145** and moreover the long code can be complex with the real multiply **145** replaced by the equivalent complex multiply **143**.

It should be obvious to anyone skilled in the communications art that this example implementation clearly defines the fundamental current CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

FIG. **4A** multiple data rate real Walsh CDMA decoding is a representative implementation ~~of~~ algorithm for the multiple data rate real Walsh CDMA decoding **27** in FIG. **3A, 144,145** in FIG. **3B**, and in equations **(2)**. Inputs are the received estimates of the

multiple data rate comple real Walsh CDMA encoded chips $\{\hat{Z}(n)\}$ **32**. The PN ~~scrambling~~ codes ~~is~~ are stripped off from these chips **33** by implementing ~~changing the sign of each chip according to the numerical sign of the real and imaginary components of the complex conjugate of the PN code as per~~ the decoding algorithms **8** in equations **(2)**. Real Walsh channelization coding is removed in **34** by a pulse compression operation consisting of multiplying each received chip by the numerical sign of the corresponding Walsh chip for the user and summing the products over the N Walsh chips to recover estimates $\{\hat{Z}(u_{m,k_m})\}$ **35** of the user complex data symbols $\{Z(u_{m,k_m})\}$.

It should be obvious to anyone skilled in the communications art that this example implementation clearly defines the fundamental current CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

For cellular applications the transmitter description describes the transmission signal processing applicable to this invention for both the hub and user terminals, and the receiver describes the corresponding receiving signal processing for the hub and user terminals for applicability to this invention.

For optical communications applications the the microwave processing at the front end of both the transmitter and the receiver is replaced by the optical processing which performs the complex modulation for the optical laser transmission in the transmitter and which performs the optical laser receiving function of the microwave processing to recover the complex baseband received signal.

Complex Walsh codes have been proposed during the early work on Walsh bases and codes, based on the even and odd sequency property of the Walsh bases and their correspondence with the even cosine real components and odd sine imaginary components of the DFT (Discrete Fourier Transform). Sequency for the Walsh is the average rate of phase rotations and is the Walsh equivalent of the frequency rotation for the Fourier and DFT bases. Walsh bases are re-ordered Hadamard bases where the ordering corresponds to increasing sequency. Gibbs in the 1970 report "Discrete Complex Walsh Sequences" developes a complex Walsh basis (each basis vector is a complex orthogonal CDMA code) from the real Walsh with the property that similar to the DFT the real part is an even function and the imaginary part is an odd function and takes the values {1,j,-1,-j}. Ohnsorg et. al. in the 1970 report "Application of Walsh Functions to Complex Signals" developed a complex Walsh basis from the real Walsh by generating a complex binary matrix from the Hadamard representation with values {1,j.-1.j} and combining the scaled sum and differences of this matrix to form a complex Walsh matrix of basic vectors which gives this matrix the real even and imaginary odd properties of the DFT. These complex Walsh bases have had no apparent value in signal processing since they were not derived as an isomorphic mapping from the DFT and therefore do not exhibit any of the DFT performance advantages over the real Walsh and moreover do not have simple and fast algorithms for coding and decoding and as a result they have not been used for CDMA communications.

Yang (US 6,674,712) combines the quaternary complex-valued Kerdock codes with the real Walsh codes to generate a set of quasi-orthogonal CDMA codes using the complex multiply operation **126** in FIG. **1C** to combine the real Walsh codes **120, 121** with the complex Kerdock codes upon replacing the complex short PN codes **124, 125** with the Kerdock codes, adding a zero to the Kerdock codes of length (2^K-1) to make them 2^M chip codes and using

real Walsh 2^M chip codes, to allow the phase addition of these codes in the complex multiply **126**. Prior art represented by the paper by Hannon et. al. (IEEE Trans. Inform. Theory, vol. 40, pp. 301-319, 1994) and other prior publications derived the Kerdock codes with the permutation and construction algorithm in this patent. Unlike Yang, current CDMA art uses the same 2^M PN code for all real Walsh channelization codes which keeps the orthogonality property while providing the desired low correlation sidelobe properties.

Honkasalo (US-6,317,413) develops a method to assign Walsh codes to variable data rate users for CDMA communications which is an application of the current OVSF in equations **(1),(2)** and in FIG. **2B** to the cellular network example in FIG. **1B** for the link **106** between the mobile user **105** and base station **107**. In the example Tx implementation for the fundamental and supplementary users, there are N_4=2^4=16 channels available at the highest data rate R supported by the communications link. Each channel is encoded with a 1x16 chip Walsh code selected from the 16x16 Walsh code matrix W_4. To support R and lower data rates R/2,R/4,R/8,R/16 and allow several users to occupy each channel, the user code lengths are extended to 1xN_5, 1xN_6, 1xN_7, 1xN_8=2^8=256 chips respectively as shown in equations **(1)**. From equations **(1),(2)** the code index c for the lowest data rate can be written as the binary word $c=c_0c_1c_2c_3c_4c_5c_6c_7$ where the $c_1,\ldots,c_8$ are the binary coefficients. The first 4 bits $c_0c_1c_2c_3$ are the W_4 code for users at rate R, the first 5 bits $c_0c_1c_2c_3c_4$ are the W_5 code index for users at data rate R/2, . . ., and the 8 bit word $c_0c_1c_2c_3c_4c_5c_6c_7$ is the W_8 code index for the lowest data rate R/16. This enables the code assignments to be specified by the 4 bit subfield $c_0c_1c_2c_3$ of c for the 16 channels and the last 4 bits $c_4c_5c_6c_7$ for the lower user data rates. Knowing the channel assignment this allows the users within a channel to be specified by the last 4 bits.

Prior art in the vol. 27 November 1973 Archive fur Elektronik und Uebertragungsteckhik paper "Aufbau und Eigenschaften von quasiothogonalen Codekollektiven" and in the 1981 Lincoln Lab. report IFF-7 introduced the concept of covering the real Walsh encoded data with a real PN code in order to improve the correlation performance with time and frequency offsets. This concept was introduced well in advance of it's use in the late 1980's introduction of CDMA (US 5,103,459) wherein the real Walsh encoded data is covered by a real PN code and which covering was later updated using a complex PN code depicted in **24,25,26** FIG. **1C** and decovered in **41,42,43** FIG.**3B.**

SUMMARY OF THE INVENTION

~~**SUMMARY OF INVENTION**~~

~~This~~ The present invention provides a method and system for multiple data rate fast encoding and fast decoding of Hybrid Walsh codes and generalized Hybrid Walsh codes for use in CDMA communications as the orthogonal channelization codes to replace the real Walsh codes. Hybrid Walsh codes generated in this invention disclosure are complex Walsh codes that have an

isomorphic one-to-one correspondence with the discrete Fourier transform (DFT) codes. Additionally, the encoding (covering) of the Hybrid Walsh complex code by a complex PN code is a novel idea introduced in this invention disclosure.

~~is a new set of fast and computationally efficient algorithms for new multiple data rate orthogonal channelization encoding and decoding for CDMA using the new complex Walsh codes and the hybrid complex Walsh orthogonal codes in place of the current real Walsh orthogonal codes. Real Walsh codes are used for current CDMA applications and will be used for all of the future CDMA systems. The newly invented complex Walsh codes disclosed in [6] provide the choice of using the new complex Walsh codes or the real Walsh codes since the real Walsh codes are the real components of the complex Walsh codes. This means an application capable of using the complex Walsh codes can simply turn-off the complex axis components of the complex Walsh codes for real Walsh CDMA coding and decoding.~~

Hybrid Walsh codes are the closest possible approximation to the DFT with orthogonal code vectors taking the values {1+j, -1+j, -1-j, 1-j} or equivalently the values {1, j, -1, -j} when the axes are rotated and renormalized and Hybrid Walsh codes offer performance improvements over real Walsh codes for CDMA communications. Hybrid Walsh codes are derived by separate lexicographic reordering permutations with increasing sequency of real Walsh codes for the inphase (real) components and for the quadrature (imaginary) components.

The invention discloses a method and system for the Hybrid Walsh encoder and decoder to be generalized by combining with DFT, Hadamard, and other codes using tensor product construction, direct sum construction, and functional combining. This construction for generalized Hybrid Walsh codes increase the choices for the code length by allowing the combined use of complex Walsh with lengths 2^M and 4t where M and t are integers,

with DFT complex orthogonal codes with lengths N where N is an integer, with Hadamard codes, with quasi-orthogonal PN families of codes including segments of maximal-length shift register codes, Gold, Kasami, Golay, Kerdock, Preparate, Goethals, STC, and with other families of codes.

The invention provides a method and system for implementing simultaneous multiple data rate users with variable code sets assigned to multiple data rate users and with the capability to be assigned to different sequency spectrums analogous to frequency division multiplexing (FDM). Additional advantages compared to OVSF are the added performance improvements that will be realized by using the codes disclosed in this invention in place of the real Walsh codes and from the greater number of choices for the code lengths available compared to real Walsh codes.

This invention provides a method and system for the fast and computationally efficient encoding and decoding of the Hybrid Walsh and generalized Hybrid Walsh code for multiple data rates.

This invention offers a method and system for providing the current and future applications of real Walsh channelization codes for CDMA with the option of using the Hybrid Walsh and the generalized Hybrid Walsh codes. An application can simply turn-off the complex axis components of the Hybrid Walsh codes and the generalized Hybrid Walsh codes and thereby reduce the signal processing to the real Walsh or equivalently the real Hadamard codes along the inphase and quadrature axes.

~~Performance is improved for the multiple data rate CDMA communications when the new 4-phase complex Walsh orthogonal CDMA codes replace the current 2-phase real Walsh codes. These~~

~~improvements include an increase in the carrier-to-noise ratio (CNR) for data symbol recovery in the receiver, lower correlation side-lobes under timing offsets both with and without PN spreading, lower levels of harmonic interference caused by non-linear amplification of multi-carrier CDMA signals, and reduced phase tracking jitter for code tracking to support both acquisition and synchronization. These potential performace improvements simply reflect the widely known principle that complex CDMA is better than real CDMA.~~

~~In addition to the performance improvement, there are greater code length choices for multiple data rate CDMA communications using the new hybrid complex Walsh orthogonal CDMA codes in place of the complex Walsh orthogonal CDMA codes which have been disclosed in [6]. Code length choices are increased by the combined use of complex Walsh and discrete Fourier transform complex orthogonal codes using a Kronecker construction, direct sum constructioin, as well as the possibility for more general functional combining.~~

BRIEF DESCRIPTION OF THE DRAWINGS AND THE PERFORMANCE DATA

~~**BRIEF DESCRIPTION OF DRAWINGS**~~

The above-mentioned and other features, objects, design algorithms, and performance advantages of the present invention will become more apparent from the detailed description set forth below when taken in conjunction with the drawings and performance data wherein like reference characters and numerals denote like elements, and in which:

FIG. **1A** is a representative CDMA transmitter signal processing implementation block diagram with emphasis on the current multiple data rate real Walsh CDMA encoding ~~which~~ and on

~~contains~~ the signal processing elements addressed by this invention disclosure.

FIG. **1B** is a schematic CDMA cellular network with the communications link between a base station and one of the multiple users.

FIG. **1C** depicts the transmit real Walsh CDMA encoding signal processing implementation for the forward and reverse links between the base station and the multiple data rate users in the cellular network.

FIG. **1D** defines the implementation algorithm of this invention disclosure for generating Hybrid Walsh codes from real Walsh.

FIG. **1E** is an embodiment of this invention disclosure for the transmit CDMA encoding signal processing implementation for the cellular network using Hybrid Walsh codes in place of real Walsh codes for the forward and reverse links between the base station and multiple data rate users.

FIG. **2A** is a representative multiple data rate real Walsh CDMA encoding implementation diagram ~~with emphasis on the current multiple data rate real Walsh CDMA encoding~~ which contains the signal processing elements addressed by this invention disclosure.

FIG. **2B** is a representative multiple data rate Hybrid Walsh CDMA encoding implementation diagram which contains the signal processing elements addressed by this invention disclosure.

FIG. **3A** is a representative CDMA receiver signal processing implementation block diagram with emphasis on the current

multiple data rate real Walsh CDMA decoding ~~which contains~~and on the signal processing elements addressed by this invention disclosure.

FIG. **3B** is a representative real Walsh CDMA decoding signal processing implementation for the forward and reverse links between the base station and the multiple data rate users in the cellular network.

FIG. **3C** is an embodiment of of this invention disclosure for the receive CDMA decoding signal processing implementation for the cellular network using Hybrid Walsh codes in place of real Walsh codes for the forward and reverse links between the base station and the mltiple data rate users.

FIG. **4A** is a representative CDMA decoding implementation diagram ~~with emphasis on the current~~for multiple data rate real Walsh CDMA decoding which contains the signal processing elements addressed by this invention disclosure.

FIG. **4B** is a representative CDMA decoding implementation diagram for multiple data rate Hybrid Walsh CDMA decoding which contains the signal processing elements addressed by this invention disclosure.

FIG. **5A** is a representative CDMA encoding implementation diagram which describes the ~~new complex~~generalized Hybrid Walsh ~~and hybrid complex Walsh~~ CDMA fast encoding of multiple data rate users and which contains the signal processing elements addressed by this invention disclosure.

FIG. **5B** is a representative CDMA encoding implementation diagram which describes the Hybrid Walsh CDMA fast encoding of multiple data rate users and which contains the signal processing elements addressed by this invention disclosure.

FIG. **6A** is a representative CDMA decoding implementation diagram which describes the ~~new complex~~genralized Hybrid Walsh ~~and hybrid complex Walsh~~ CDMA fast decoding of multiple data rate users and which contains the signal processing elements addressed by this invention disclosure.

FIG. **6B** is a representative CDMA decoding implementation diagram which describes the Hybrid Walsh CDMA fast decoding of multiple data rate users and which contains the signal processing elements addressed by this invention disclosure.

DISCLOSURE OF THE INVENTION

~~**DISCLOSURE OF INVENTION**~~

The ~~new~~ invention provides the algorithms and implementation architectures to support simultaneous multiple data rates or equivalently simultaneous multiple symbol rates using the ~~new complex~~Hybrid Walsh and generalized ~~h~~Hybrid ~~complex~~ Walsh orthogonal CDMA codes. ~~which have been disclosed in the invention application [6]. Simultaneous multiple data rates over the same CDMA frequency spectrum are well known in CDMA networking and been included in the next generation UMTS 3G evolving CDMA using wideband CDMA (W-CDMA) and real Walsh orthogonal CDMA channeliztion codes.~~

~~The current~~ In contrast to current art which ~~uses three categories of~~assigns multiple length real Walsh codes to the

multiple data rate users with the shorter codes assigned to the higher data rate users, ~~techniques designed to accommodate multiple data rate users and these are A) multiple chip length codes for the multiple data rate users, B)~~ the invention uses same chip length codes with the number of codes adjusted as required for the multiple data rate users~~, and C)~~ and also has the ability to assign different sequency spectrums to each data rate group of users. This invention supports fast (efficient) encoding and decoding implementations.~~different frequency spectrums assigned to the multiple data rate users which is frequency division multiplexing (FDM). The first technique is the preferred choice for W-CDMA primarily because of the de-multiplexing and multiplexing required for the second technique and because of the configurable multi-rate filters required for the spectrum partitioning in the third approach. This new invention implements the second and third approaches without their disadvantages and moreover provides the added~~ and provides ~~performance~~ improvements ~~that will be realized~~ with the use of the ~~complex~~ Hybrid Walsh and generalized ~~h~~Hybrid ~~complex~~ Walsh codes in place of the real Walsh codes. ~~These new~~Hybrid Walsh codes are 4-phase complex ~~Walsh~~ orthogonal CDMA codes ~~replacing~~ to replace the current 2-phase real Walsh codes ~~will~~ and to provide improvements that include an increase in the carrier-to-noise ratio (CNR) for data symbol recovery in the receiver, lower correlation side-lobes under timing offsets both with and without PN spreading, lower levels of harmonic interference caused by non-linear amplification of multi-carrier CDMA signals, and reduced phase tracking jitter for code tracking to support both acquisition and synchronization. These potential performace improvements simply reflect the widely known principle that complex CDMA is better than real CDMA. The generalized ~~h~~Hybrid ~~complex~~ Walsh offers these same improvements together with the flexibility of more choices in the code lengths at the expense of increasing the number of code phases on the unit circle thereby

introducing multiplications into the encoding and decoding implementations.

1. Hybrid Walsh Encoder and Decoder

The ~~new complex~~Hybrid Walsh ~~and hybrid complex~~ Walsh CDMA orthogonal codes ~~disclosed in [6]~~ have been invented to be the natural ~~development~~extension real ~~for~~ of the Walsh codes to the complex domain. ~~and therefore are the correct complex Walsh codes to within arbitrary factors that include scale and rotation, which are not relevant to performance. This~~ These Hybrid Walsh ~~natural development of the complex Walsh~~ codes in the N-dimensional complex code space C^N ~~extended~~ are the extension of the 1-to-1 correspondence~~s~~ between the real Walsh codes and the Fourier codes in the N-dimensional real code space R^N, to the 1-to-1 correspondence~~s~~ between the complex Walsh codes and the ~~discrete Fourier transform (~~DFT~~)~~ codes in C^N.

Equations **(3)** define the DFT complex codes in C^N as a function of the real Fourier codes in R^N. These results together with the correspondence between the Hybrid Walsh and the DFT codes will enable the Hybrid Walsh codes in C^N to be derived as a function of the real Walsh codes in R^N in equations (21). The NxN matrices $F, E, W, \widetilde{W}$ are the respective code matrices for the sets of Fourier, DFT, Walsh, Hybrid Walsh codes in R^N, C^N, R^N, C^N and are constructed with the row code vectors $\{F(c)\}$, $\{E(c)$ $, (W(c)\}, \{\widetilde{W}(c)\}$. Each code vector is a 1xN vector code sequence with component values on the unit circle. Decoding from a matrix viewpoint is the multiplication of the NxN code matrix with the conjugate transpose of the NxN code matrix. In **401** the real even cosine code vectors $\{C(c)\}$ and odd sine code vectors $\{S(c)\}$ are defined as the real and imaginary components of $\{E(c)\}$ in C^N. The set of Fourier codes in R^N is the N-code subset **402**

of these cosine and sine codes which span R^N. This set of Fourier codes can be used to define the DFT codes **403** by applying the DFT spectral foldover property which observes the DFT harmonic vectors for frequencies $fNT=N/2+\Delta u$ above the half-Nyquist sampling rate $fNT=N/2$ simply foldover such that the DFT harmonic vector for $fNT = N/2+\Delta u$ is the DFT basis vector for $fNT = N/2-\Delta u$ to within a fixed sign and fixed phase angle of rotation. and wherein "f" is the frequency and "T" is the discrete sampling interval. From a mathematical viewpoint, the DFT codes in **403** can be equivalently defined by using the trigonometric identities $C(N/2+\Delta c)=C(N/2-\Delta c)$ and $S(N/2+\Delta c)=(-)S(N/2-\Delta c)$ together with the Fourier codes **402.**

DFT codes in C^N derived from Fourier in R^N **(3)**

401 DFT codes in C^N

E = DFT NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [E(c)] matrix of row vectors E(c)

= [E(c,n)] matrix of elements E(c,n)

$E(c) = C(c) + j\ S(c)$ for $c=0,1,\ldots,N-1$

where

$C(c)$ = Even code vectors for $c=0,1,\ldots,N-1$

$= [1, \cos(2\pi\ c\ 1/N),\ldots,\cos(2\pi\ c\ (N-1)/N)]$

$S(c)$ = Odd code vectors for $c=0,1,\ldots,N-1$

$= [0, \sin(2\pi\ c\ 1/N),\ldots,\sin(2\pi\ c\ (N-1)/N)]$

$E(c,n)$ = DFT code c chip n

$= e^{\wedge}(j2\pi cn/N)$

$= \cos(2\pi\ cn/N) + j\ \sin(2\pi\ cn/N)$

402 Fourier codes in R^N

Fourier codes code set are the N codes:

Even codes { $C(c)$, $c=0,1,2,\ldots,N/2$

Odd codes{ $S(c)$, $c= 1,2,\ldots,N/2-1$

403 DFT codes derived from Fourier codes

for $c = 0,1,\ldots,N/2$

$$E(c) = C(0) \qquad \text{for } c = 0$$
$$= C(c)+ j\, S(c) \qquad \text{for } c = 1,2,\ldots,N/2-1$$
$$= C(N/2) \qquad \text{for } c = N/2$$

for $c = N/2+1,\ldots,N-1$

$$= N/2 + \Delta c \quad \text{with } \Delta c = 1,\ldots,N/2-1$$

$$E(c) = C(N/2 -\Delta c) - j\, S(N/2-\Delta c)$$

Equations **(4)** derive the Hybrid Walsh codes in C^N as lexicographic reordering permutations of the real Walsh codes in R^N by combining the 1-to-1 correspondence of the real Walsh codes with the Fourier, the 1-to-1 correspondence of the Hybrid Walsh codes with the DFT, and the derivation of the DFT codes in C^N as a function of the Fourier codes in R^N in equations **(3)**. In equations **(4)** the even and odd real Walsh codes in **404** are placed in a 1-to-1 correspondence with the cosine and sine Fourier codes in **405** wherein the 1-to-1 correspondence is indicated by the symbol "~" and the correspondence is in lexicographic ordering with increasing sequency and frequency such that "sequency~frequency" meaning that sequency in the real Walsh domain corresponds to frequency in the Fourier domain. In this invention disclosure the Hybrid Walsh is derived as a unique 1-to-1 correspondence between the Hybrid Walsh codes and the DFT in **407**. The derivation in **407** starts with the Hybrid Walsh definition in **406**. Next, the Hybrid Walsh Next, the Hybrid Walsh

codes are defined in in **406** by combining **403** in equations **(3)** with **404**,**405**,406.

Hybrid Walsh codes in C^N derived from real Walsh in R^N **(4)**

404 Even and odd real Walsh codes in R^N

$W_e(u)$ = Even Walsh code vector

= $W(2u)$ for $u=0,1,\ldots,N/2-1$

$W_o(u)$ = Odd Walsh code vectors

= $W(2u-1)$ for $u=1,\ldots,N/2$

where W_e, W_o are even,odd real Walsh codes

405 Correspondence between real Walsh and Fourier in R^N

$W(0) \sim C(0)$

$W_e(c) \sim C(c)$ for $c = 1,\ldots,N/2-1$

$W_o(c) \sim S(c)$ for $c = 1,\ldots,N/2-1$

$W(N-1) \sim C(N/2)$

where "~" represents a 1-to-1 correspondence

406 Hybrid Walsh $\tilde{W}(c) = W(cr) + j\,W(ci)$ in C^N

$cr = cr(c)$

= lexicographic reordering permutation for the real components of the Hybrid Walsh codes

$ci = ci(c)$

= lexicographic reordering permutation for the imaginary components of the Hybrid Walsh codes

Correspondence between Hybrid Walsh and DFT

$\tilde{W}(c) \sim E(c)$ for $c=0,1,2,\ldots,N-1$

Definition of the Hybrid Walsh codes:

for $c = 0$

$\tilde{W}(c) = W(0) + jW(0) \sim E(c) = 1$

for $c = 1,2,\ldots,N/2-1$

$W(cr) = W_e(c) = W(2c) \sim C(c) = \mathrm{Real}\{E(c)\}$

$W(ci) = W_o(c) = W(2c-1) \sim S(c) = \mathrm{Imag}\{E(c)\}$

for $c = N/2$

$\tilde{W}(c) = W(N-1)+j\,W(N-1) \sim E(c) = C(N/2)$

for $c = N/2+\Delta c$, $\Delta c=1,2,\ldots,N/2-1$

$W(cr) = W(N-1-2\Delta c) \sim C(N/2-\Delta c)=\mathrm{Real}\{E(c)\}$

$W(ci) = W(N-1-(2\Delta c-1))$

$= W(N-2\Delta c) \sim S(N/2-\Delta c) = (-)\mathrm{Imag}\{E(c)\}$

An equivalent way to derive the complex Hybrid Walsh code vectors in C^N from the real Walsh basis in R^{2N} is to use a sampling technique which is a known method for deriving a complex DFT basis in C^N from a Fourier real basis in R^N.

FIG. **1D** summarizes the Hybrid Walsh implementation algorithms derived in equation **(21)** for implementation as lexiocographic reordering permutations of the real Walsh code vectors with the reordering lexicographically arranged with increasing sequency in agreement with the correspondence "sequency ~ frequency" for "Hybrid Walsh ~ DFT". The real axis (inphase) reordering permutation **168** in FIG. **1D** is implemented as an address change $cr=cr(c)$ of the row vectors in W to define the row vectors $W(cr)$ of the real code components of $\tilde{W}(c)$ in lexicographic ordering with increasing sequency **167**. Likewise,

the imaginary (quadrature) reordering permutation **169** is defined as an address change ci=ci(c) of the row vectors in W to correspond to the row vectors W(ci) of the imaginary code components of $\tilde{W}(c)$ in lexicographic ordering with increasing sequency **167**. These reordering permutations define the Hybrid Walsh $\tilde{W}(c)$ = W(cr)+jW(ci).

FIG. **1E** is the upgrade to the cellular network transmit CDMA encoding in FIG. **1B** using the Hybrid Walsh channelization codes in place of the real Walsh codes. FIG. **1E** depicts a representative embodiment of the transmitter signal processing for the forward and reverse CDMA links **106** in FIG. **1B** between the base station and the user for CDMA2000 and W-CDMA. Similar to FIG. **1C** the data inputs are the inphase data symbols R **173** and quadrature data symbols I **174**. Inphase **175** Hybrid Walsh codes W(cr) are implemented in FIG. **1D 167,168** and in equations **(3)**. Quadrature **176** Hybrid Walsh codes W(ci) are implemented in FIG. **1D 167,169** and in equations **(3)**. A complex multiply **177** encodes the data symbols with the Hybrid Walsh $\tilde{W}$ codes in the encoder using the inphase (real) W(cr) and quadrature (imaginary) W(ci) code components of $\tilde{W}(c)$ = W(cr)+j W(ci) to generate a rate R=N set of Hybrid Walsh encoded data chips for each inphase and quadrature data symbol. Following the Hybrid Walsh encoding the transmit signal processing in **178-to-189** is identical to the corresponding transmit signal processing in **122-to-133** in FIG. **1C**.

FIG. **1E** depicts an embodiment of the upgrade to the current CDMA transmitter art using the Hybrid Walsh codes in place of the real Walsh codes and with current art signal processing changes this figure is representative of the use of Hybrid Walsh codes in place of the real Walsh codes for other current art CDMA receiver embodiments of this invention disclosure. Other embodiments of

the CDMA transmitter include changes in the ordering of the signal processing, single channel versus multi-channel Hybrid Walsh encoding, summation or combining of the Hybrid Walsh channels by summation over like chip symbols, analog versus digital signal representation, baseband versus IF frequency CDMA processing, the order and placement in the signal processing thread of the Σ, LPF, and D/A signal processing operations, and the up-conversion processing. The order of the rate R=1 PN code multiplies in FIG. **1E** can be changed since the covering operations implemented by the multiplies are linear in phase, which means the short code complex multiply **180,181,182** in FIG. **1E** can occur prior to the long code multiply **178,179** and moreover the long code can be complex with the real multiply **179** replaced by the equivalent complex multiply **182.**

FIG. **3C** is the upgrade to the cellular network receive CDMA decoding in FIG. **3B** using the Hybrid Walsh complex channelization codes in place of the real Walsh codes. FIG. **3C** depicts a representative embodiment of the receiver signal processing for the forward and reverse CDMA links **106** in FIG. **1B** between the base station and the user for CDMA2000 and W-CDMA that implements the CDMA decoding for the decovering by the long code and the short complex codes followed by the Hybrid Walsh decoding to recover estimates of the transmitted inphase (real) data symbols R **173** and quadrature (imaginary) data symbols I **174** in FIG. **1E.** Depicted are the principal signal processing that is relevant to this invention disclosure. Signal input $\hat{v}(t)$ **190** is the received estimate of the transmitted CDMA signal v(t) **189** in FIG. **1E.** The receive signal recovery in **191-to-201** is identical to the corresponding receive signal processing in **135-to-145** in FIG. **3B.** The decovered chip symbols are rate R=1/N decoded by the Hybrid Walsh complex decoder **204** using the complex conjugate of the Hybrid Walsh code structured as the inphase Hybrid Walsh code W_R **202** and the negative of the quadrature Hybrid Walsh code $(-)W_I$

203 to implement the complex conjugate of the Hybrid Walsh code in the complex multiply and decoding operations. Decoded output symbols are the inphase data symbol estimates $\hat{R}$ **205** and the quadrature data symbol estimates $\hat{I}$ **206**.

FIG. **3C** depicts an embodiment of the upgrade to the current CDMA receiver art using the Hybrid Walsh code in place of the real Walsh code and with current art signal processing changes this figure is representative of the use of Hybrid Walsh codes in place of the real Walsh codes for other current art CDMA receiver embodiments of this invention disclosure. Other embodiments of the CDMA receiver include changes in the ordering of the signal processing, analog versus digital signal representation, down-conversion processing, baseband versus IF frequenncy CDMA processing, the order and placement in the signal processing thread of the Σ, LPF, and A/D signal processing operations, and single channel versus multi-channel Hybrid Walsh decoding, The order of the rate R=1 PN code multiplies in FIG. **7B** which perform the code decovering can be changed since the covering operations implemented by the multiplies are linear in phase, which means the short code complex multiply **197,198,199** can occur after to the long code multiply **200,201** and moreover the long code can be complex with the real multiply **201** replaced by the equivalent complex multiply **199**.

2. Generalized Hybrid Walsh Codes

The ~~new~~ generalized ~~hybrid~~ Hybrid ~~complex~~ Walsh orthogonal CDMA codes increase the choices for the code length by allowing the combined use of ~~complex~~ Hybrid Walsh and ~~discrete Fourier~~DFT ~~transform complex~~ orthogonal codes using a ~~Kronecker~~ tensor

product construction, direct sum constructioin, as well as the possibility for more general functional combining including the use of PN codes. Generalized Hybrid ~~complex~~ Walsh orthogonal CDMA codes increase the flexibility in choosing the code lengths for multiple data rate users at the implementation cost of introducing multiply operations into the CDMA encoding and decoding~~.~~ or degrading the orthogonlity property to quasi-orthogonality. ~~Two of several means for construction given in the patent application [6] are the Kronecker product and the direct sum. The direct sum will not be considered since the addition of the zero matrix in the construction is generally not desirable for CDMA communications although the direct sum construction provides greater flexibility in the choice of N without necessarily introducing a multiply penality. Using the Kronecker product construction in reference [6] the hybrid complex Walsh orthogonal CDMA codes can be constructed as demonstrated in equations (4).~~

Equations **(5)** list construction and examples of the generalizd ~~h~~Hybrid ~~complex~~ Walsh orthogonal CDMA codes using the ~~Kronecker~~ tensor product ~~approach and~~with the NxN DFT matrices E_N and Hybrid Walsh matrices $\widetilde{W}_N$ and functional combining with direct sums. ~~to expand the complex Walsh to a hybrid complex Walsh.~~ Low order CDMA code examples ~~**45**~~ **41** illustrate fundamental relationships between the DFT, ~~complex~~ Hybrid Walsh, and the real Walsh or equivalently Hadamard. ~~Kronecker~~ Tensor product construction is defined in ~~**46**~~**42**. CDMA current and developing standards use the prime 2 which generates a code length $N=2^M$ where M=integer. For applications requiring greater flexibility in code length N, additional primes can used using the ~~Kronecker~~ tensor product construction. We illustrate this in the examples ~~**47**~~ **43** with the ~~addition~~ use of prime=3. The use of prime=3 in addition to the prime=2 in the range of N=8 to 64 is observed to increase the number of N choices from 4 to 9 at a

modest cost penality of using multiples of the angle increment 30 degrees for prime=3 in addition to the angle increment 90 degrees for prime=2. As noted in ~~**46**~~ **43** there are several choices in the ordering of the ~~Kronecker~~ tensor product construction and 2 of these choices are used in the construction. In general, the orthogonal code matrices are dependent on the ordering of the tensor product which means different orderings produce different orthogonal code matrices. Direct sum construction provides greater flexibility in the choice of N without necessarily introducing a multiply penality. However, the addition of the zero matrix in the construction is generally not desirable for CDMA communications. A functional combining in **44** in equation **(5)** removes these zero matrices at the cost of relaxing the orthogonality property to quasi-orthogonality.

~~Examples of~~ Generalized ~~h~~Hybrid ~~complex~~ Walsh ~~orthogonal~~ code~~s~~ construction ____________________ **(5)**

~~**45**~~ **41** Examples of low-order codes

2x2 $E_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$= (e^{-j\pi/4}/\sqrt{2}) * \widetilde{W}_2$

$= H_2$ 2x2 Hadamard

3x3 $E_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{j2\pi/3} & e^{j2\pi 2/3} \\ 1 & e^{j2\pi 2/3} & e^{j2\pi/3} \end{bmatrix}$

4x4 $\widetilde{W}_4 = \begin{bmatrix} 1+j & 1+j & 1+j \\ 1+j & -1+j & -1-j & 1-j \\ 1+j & -1-j & 1+j & -1-j \\ 1+j & 1-j & -1-j & -1+j \end{bmatrix} 1+j$

$$E_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= (e^{-j\pi/4}/\sqrt{2})\ \tilde{W}_4$$

4~~6~~2 ~~Kronecker~~ Tensor product construction for $N = \prod_k N_k$

Code matrix C_N = NxN ~~hybrid~~ generalized Hybrid Walsh orthogonal CDMA code matrix

~~Kronecker~~ Tensor product construction of C_N

$$C_N = C_0 \prod_{k>0} \otimes C_{N_k}$$

where C_0, C_{N_k} are DFT, Hybrid Walsh code matrices

~~Kronecker~~ Tensor product definition

A = $N_a x N_a$ orthogonal code matrix

B = $N_b x N_b$ orthogonal code matrix

$A \otimes B$ = ~~Kronecker~~ Tensor product of matrix A and matrix B

= $N_a N_b$ x $N_a N_b$ orthogonal code matrix consisting of the elements $[a_{ik}]$ of matrix A multiplied by the matrix B

= $[\ a_{ik}\ B\ ]$

4~~7~~3 ~~Kronecker~~ Tensor product construction examples for primes p=2,3 and the range of sizes $8 \le N \le 64$

8x8 $C_8 = \tilde{W}_8$

12x12 $C_{12} = \tilde{W}_4 \otimes E_3$

	C_{12}	=	$E_3 \otimes \widetilde{W}_4$
16x16	C_{16}	=	$\widetilde{W}_{16}$
18x18	C_{18}	=	$\widetilde{W}_2 \otimes E_3 \otimes E_3$
	C_{18}	=	$E_3 \otimes E_3 \otimes \widetilde{W}_2$
24x24	C_{24}	=	$\widetilde{W}_8 \otimes E_3$
	C_{24}	=	$E_3 \otimes \widetilde{W}_8$
32x32	C_{32}	=	$\widetilde{W}_{32}$
36x36	C_{36}	=	$\widetilde{W}_4 \otimes \widetilde{W}_3 \otimes \widetilde{W}_3$
	C_{36}	=	$\widetilde{W}_3 \otimes \widetilde{W}_3 \otimes \widetilde{W}_4$
48x48	C_{48}	=	$\widetilde{W}_{16} \otimes \widetilde{W}_3$
	C_{48}	=	$\widetilde{W}_3 \otimes \widetilde{W}_{16}$
64x64	C_{64}	=	$\widetilde{W}_{64}$

44 Generalized Hyhrid Walsh quasi-orthogonal code matrices using functional combining with direct sum construction for $N=\sum_k N_k$

Code matrix C_N = NxN generalized Hybrid Walsh quasi-orthogonal Walsh CDMA code matrix using functional combing with direct sum construction of C_N

$$C_N = f(\ C_0 \prod_{k>0} \oplus C_{N_k}\ ,\ C_P\ )$$

wherein

A = $N_a x N_a$ orthogonal code matrix

B = $N_b x N_b$ orthogonal code matrix

A $\oplus$ B = Direct sum of matrix A and matrix B

= N_a+N_b x N_a+N_b orthogonal code matrix

$$= \left[\begin{array}{c:c} A & O_{N_a x N_b} \\ \hdashline O_{N_b x N_a} & B \end{array}\right]$$

$O_{N_1 x N_2}$ = $N_1 x N_1$ zero matrix

f(A,b) = functional combining operator of A,B

= the element-by-element covering of A with B for the elements of A $\neq$ 0,

= the element-by-element sum of A and B for the elements of A = 0

C_P = NxN pseudo-orthogonal complex code matrix whose row code vectors are independent strips of PN codes for the real and imaginary components

3. Multiple Data Rate Hybrid Walsh Encoder and Decoder

Transmitter equations **(~~3~~6)** describe ~~a~~ representative ~~complex~~ Hybrid Walsh CDMA encoding and decoding algorithms for multiple data rate users for implementation in the transmitters in FIG. **1A** and FIG. **1E** and in the receivers in FIG. **3A** and FIG. **3C** assuming that the data symbols $Z(u_{m,k_m})$ in **3** in equations **(1)**, in **17** in FIG. **2A**, and in **412-416** in FIG. **2C** have already been formatted or equivalently mapped into the data symbol vector Z(c) for Hybrid Walsh encoding and which mapping from a hardware implementation is a memory (Mem) store and multiplex (Mux) set

of operations. ~~using the definition of the complex Walsh CDMA codes in the invention application [6]. Lowest data rate users are assumed rate equal to the code repetition rate of the N chip complex Walsh code, which means they are assigned N chip code vectors from the NxN complex Walsh code matrix~~ ~~$\tilde{W}_N$~~ ~~in **36** for their channelization codes. Higher data rate users will use shorter complex Walsh codes. Reference complex Walsh code matrix~~ ~~$\tilde{W}_N$~~ ~~has N Walsh row code vectors~~ ~~$\tilde{W}_N(c)$~~ ~~each of length N chips and indexed by c=0,1,…,N-1, with~~ ~~$\tilde{W}_N(c)=[\tilde{W}_N(c,0),\ldots,\tilde{W}_N(c,N-1)]$~~ ~~wherein~~ ~~$\tilde{W}_N(c,n)$~~ ~~is chip n of code c with the possible values~~ ~~$\tilde{W}_N(c,n)= +/-1 +/-j$.~~ ~~Complex Walsh code vectors in the N dimensional complex code space~~ ~~C^N~~ ~~are defined using the real Walsh code vectors from the N dimensional real code space~~ ~~R^N~~ ~~for the real and complex code vectors using the equation~~ ~~$\tilde{W}_N(c)=W(cr)+jW(ci)$~~ ~~where the mapping of the complex Walsh code index c into the real Walsh code indices cr and ci is defined by the mapping of c into cr(c) and ci(c) in **36**.~~

~~The multiple data rate menu in **37** lists the possible user data symbol rates~~ ~~R_s~~ ~~and the number of symbols transmitted over each N chip reference code length. User symbol rate~~ ~~$R_s=1/N(m)T$~~ ~~for the users in group m is equal to the number of user data symbols N/N(m) over the N chip code block multiplied by the symbol rate rate 1/NT of the N chip code. User data rate~~ ~~R_b~~ ~~in bits/second is equal to~~ ~~$R_b=R_s b_s$~~ ~~where~~ ~~b_s~~ ~~is the number of data bits encoded in each data symbol. Assuming a constant~~ ~~b_s~~ ~~for all of the multiple data rate users, the user data rate becomes directly proportional to the user symbol rate~~ ~~$R_b \sim R_s$~~ ~~which means the user symbol rate menu in **37** is equivalent to the user data rate menu.~~ Hybrid Walsh encoding for multiple data rate users is defined in **45** in equations **(3)** as a scalar set of equations and in **46** as an equivalent vector equation. Data inputs for the Hybrid Walsh N-chip block encoding are the 1xN data vector Z(c)

and the encoded output following PN encoding is the 1xN encoded chip vector Z(n). For the scalar equations the Z(c),Z(n) are considered to be the scalar components or elements of the vectors Z(c),Z(n) and for the vector equations these are considered to be vectors. Multiple data rate Hybrid Walsh decoding is defined in **47** as a scalar set of equations and in **48** as a vector equation.

~~Data symbol vector **38** stores the N data symbols~~ $\cancel{\{Z(u_{m,k_m})\}}$ ~~for the N chip code block in an 1xN dimensional data symbol vector indexed by~~ $\cancel{d=d_0+d_1 2+d_2 4+\ldots+d_{M-2}N/4+d_{M-1}N/2=0,1,2,\ldots,N-1}$~~, where the binary word represention is~~ $\cancel{d=d_0\cdots d_{M-1}}$ ~~and the~~ $\cancel{\{d_m\}}$ ~~are the binary coefficients. With the availability of this 1xN dimensional data symbol vector, it is observed that the real Walsh implementation for the multiple data rate users in **3** in equations (3) must assign the 2 chip data symbols~~ $\cancel{Z(u_0,z_k)}$ ~~to the~~ $\cancel{d_{M-1}}$ ~~field, the 4 chip data symbols~~ $\cancel{Z(u_1,z_k)}$ ~~to the~~ $\cancel{d_{M-1}d_{M-2}}$ ~~field, ..., and the N chip data symbols~~ $\cancel{Z(u_{M-1},z_k)}$ ~~to the~~ $\cancel{d_0\cdots d_{M-1}}$ ~~field in order to provide orthogonality between the code vectors in the different groups.~~

~~For the complex Walsh the same data assignment is used with the modification that the N/N(m) data symbols for the N(m) chip code vectors of group m assigned to data field~~ $\cancel{d_{M-m}d_{M-m+1}\bullet\bullet\bullet\bullet d_{M-1}}$ ~~of d using the real Walsh, are now mapped into N/N(m) N-chip code vectors over the same group m data field~~ $\cancel{d_{M-m}d_{M-m+1}\bullet\bullet\bullet\bullet d_{M-1}}$ ~~of d. This allows a fast algorithm to be used and uses the N chip codes over the~~ $\cancel{d_{M-m}d_{M-m+1}\bullet\bullet\bullet\bullet d_{M-1}}$ ~~field of d which field occupies the same sequency band as the frequency band~~

~~for FDM. This removes the disadvantages of using technique "B" and "C" for W-CDMA, and helps to make the complex Walsh the preferred choice compared to technique "A" which is the current art preferred choice with real Walsh.~~

~~The new invention has found a means to use the same data fields of the current W-CDMA for real Walsh, for application to the complex Walsh with the added advantages of a fast transform, simultaneous transmission of the user data symbols, and the assignment of these user data symbols~~ to ~~a~~ contiguous sequency subbands. ~~specified by the data field of d for additional isolation between users. For a fully loaded CDMA communications frequency band the N data symbols for the multiple rate users occupy the N available data symbol locations in the data symbol vector d= $d_0 \cdots d_{M-1}$. The construction of the data symbol vector is part of this invention disclosure and provides a means for the implementation of a fast complex Walsh encoding and decoding of the multiple data rate complex Walsh CDMA. Examples 1 and 2 in~~ ~~**39** and **40** illustrate representative user assignments to the data fields of the data symbol vector.~~

~~This mapping of the user data symbols into the data symbol vector is equivalent to setting c=d which makes it possible to develop the fast encoding algorithm **41**.~~

~~New multiple~~ Multiple data rate ~~complex~~ Hybrid Walsh ~~E~~encoding ~~for transmitter~~and decoding **(~~3~~6)**

45 Hybrid Walsh and PN encoding: scalar definition ~~N chip complex Walsh code~~

~~block~~ $Z(n) = \sum_{c} Z(c)\,\tilde{W}(c,n)\,P_2(n)\,[P_R(n) + j\,P_I(n)]$

= Hybrid Walsh CDMA encoded chip n

where

c=0,1,2,. . .,N-1 c

Z(c) = Data symbol c

~~$\tilde{W}$ = complex Walsh NxN orthogonal code matrix~~

~~consisting of N rows of N chip code vectors~~

~~= [$\tilde{W}_N$(c)] matrix of row vectors $\tilde{W}_N$(c)~~

~~= [$\tilde{W}_N$(c,n)] matrix of elements $\hat{W}_N$(c,n)~~

~~$\tilde{W}_N$(c) = complex Walsh code vector c~~

~~= W_N(cr) + jW_N(ci) for c=0,1,…,N-1~~

~~W_N(cr), W_N(ci) = Real Walsh 1xN code vectors cr,ci~~

~~c = 0,1,2,…,N-1~~

~~= Real Walsh code index for N chip block~~

~~= (cr,ci) Pair of real Walsh code vectors~~

~~cr=cr(c) and ci=ci(c) which are assigned to~~

~~the real and to the imaginary axes~~

~~n = 0,1,2,…,N-1~~

~~= Chip index for N chip block~~

~~Mapping of real Walsh to complex Walsh~~

~~Complex Walsh code~~	~~Real Axis real Walsh codes~~	~~Complex Axis real Walsh codes~~
~~c~~	~~cr(c)~~	~~ci(c)~~
~~0~~	~~0~~	~~0~~
~~1,2,…,N/2-1~~	~~2c~~	~~2c-1~~
~~N/2~~	~~N-1~~	~~N-1~~
~~N/2+1,…,N-1~~	~~2N-2c-1~~	~~2N-2e~~

$\tilde{W}$ (c,n) = complex Walsh code ~~u~~ c chip n

~~= +/-1 +/-j possible values~~

= W(cr,n)+j W(ci,n)

$$= (-1)\text{\^{}}\left[cr_{M-1}n_0 + \sum_{i=1}^{i=M-1}(cr_{M-1-i}+cr_{M-i})n_i\right]$$

$$+ j(-1)\text{\^{}}\left[ci_{M-1}n_0 + \sum_{i=1}^{i=M-1}(ci_{M-1-i}+ci_{M-i})n_i\right]$$

$cr = \sum_{i=0}^{i=M-1} cr_i\, 2^i$ binary representation of cr

$ci = \sum_{i=0}^{i=M-1} ci_i\, 2^i$ binary representation of ci

$c = \sum_{i=0}^{i=M-1} \mathbf{c}_i\, 2^i$ binary representation of c

$n = \sum_{i=0}^{i=M-1} n_i\, 2^i$ binary representation of n

~~=~~ $\cancel{W_N(cr) + jW_N(ci)}$ ~~for~~ $\cancel{c_m = 0, 1, \ldots, 2^m - 1}$

~~37Multiple data rate menu~~

~~Symbol rate menu for multiple data rates~~

~~Symbol rate~~		~~Symbol rate, Symbols/second~~	~~Symbols per N chips~~
~~R_s~~	~~=~~	~~1/2T~~	~~N/2~~
	~~=~~	~~1/4T~~	~~N/4~~
	~~=~~	~~1/8T~~	~~N/8~~
		⋮	⋮
	~~=~~	~~1/2NT~~	~~2~~
	~~=~~	~~1/NT~~	~~1~~

46 Hybrid Walsh and PN encoding: vector definition

$Z(n) = [\ Z(c) * \tilde{W}\ ] \ .*\ P_2\ .*\ [P_R + j\ P_I]$

= Hybrid Walsh CDMA encoded chip vector

where

$Z(n) = [Z(n=0), Z(n=1), \ldots, Z(n=M-1)]()$

$= 1xN$ row vector of encoded chips

$Z(c) = [Z(c=0), Z(c=1), \ldots, Z(c=N-1)]$

$= 1xN$ row vector of data symbols

"*" = vector and matrix multiplication

".*" = element-by-element vector and matrix multiplication

~~38Data symbol vector field indexed by $d=d_0+d_1 2+d_2 4+\ldots+d_{M-2} N/4+d_{M-1} N/2$ is partitioned into M data fields with each assigned to one group of multiple data rate users. Writing d as a binary word $d=d_0 d_1 \cdots d_{M-1}$ enables the data fields to be identified as d_{M-1}, $d_{M-1}d_{M-2}$, $d_{M-1}d_{M-2}d_{M-3}$, ..., $d_0 \cdots d_{M-1}$ which respectively are assigned to the user groups u_0, u_1, ..., u_{M-1}.~~

47 Hybrid Walsh CDMA decoding: scalar definition

$$\hat{Z}(c) = (2N)^{\wedge}(-1) \sum_{c} \hat{Z}(n) \widetilde{W}(c,n)' P_2(n) [P_R(n) + j\ P_I(n)]$$

$$= (2N)^{\wedge}(-1) \sum_{c} \hat{Z}(n)\ [\text{sign}\{W(n,cr)\} - j\ \text{sign}\{W(n,ci)\}]$$

$$* \text{sign}\{P_2(n)\} [\text{sign}\{P_R(n)\} - j\ \text{sign}\{P_I(n)\}]$$

= Receiver estimate of the Tx code symbol Z(c)

48 Hybrid Walsh CDMA decoding: vector definition

$$\hat{Z}(c) = (2N)^{\wedge}(-1)\ [\hat{Z}(n) * \widetilde{W}'] .* [P_2(n)] .* [P_R(n) + j\ P_I(n)]$$

$$= (2N)^{\wedge}(-1)\ \hat{Z}(n) * [\text{sign}\{W(n,cr)\} - j\ \text{sign}\{W(n,ci)\}]$$

$$.* [\text{sign}\{P_2(n)\}] .* [\text{sign}\{P_R(n)\} - j\ \text{sign}\{P_I(n)\}]$$

= Receiver estimate of the Tx code vector Z(c)

Equations (7) define the mapping (formatting) of he data input symbol vector for multiple data rate users into the code symbol vector. This mapping is constructed by parsing he code field of elements c into subfields which can be placed into a 1-to-1 corespondence with the user groups arranged according to data rate. This correspondence together with the arrangement of the parsing over the field of c codes defines the mapping algorithm for the multiple users and ensures that all users in the same group with the same data symbol rate will occupy the same sequency spectrum. The menu of allowable symbol rates and the corresponding user groups are defined in **2,3** in equation **(1)**. An alternate approach to mapping (formatting) is to directly assign to the data symbol vector the received data symbols from the users for transmission over the N-chip CDMA encoded block.

The data field mapping is developed by parsing (partitioning) the code field of elements {c} into subfields with each subfield assigned to the set of users transmitting at the same data symbol rate, and assigning the users to their appropriate subfield. This will enable the users within the same group to be contiguous in their Hybrid Walsh code assignments and thereby to transmit over the same sequency band. Parsing of the code field of elements c is defined in **49,50,51** in equations **(6)** for N=2^M. The code index c is expanded in the binary

representation as a function of the binary coefficients c_0, c_1, . . ., c_{M-2}, c_{M-1} and can be represented by the digital word $c=c_0c_1..c_{M-2}c_{M-1}$. The finite set of elements indexed on c is a Galois field GF(2^M) of N=2^M elements. The algorithm in **49,50,51** defines the unique parsing of the GF(2^M) into subfields for the user groups and is summarized in FIG. **2B.**

In **49** the mapping of the input data symbols $\{Z(u_m,k_m)\}$ onto the data symbol vector Z(c) is a linear transformation consisting of a data symbol store plus a multiplexing to define Z(c) and the mapping is defined in **50,51** and in FIG. **2B.** In **50** the M subfields of GF(2^M) c_{M-1}, $c_{M-2}c_{M-1}$, $c_{M-3}c_{M-2}c_{M-1}$, . . . are mapped onto the data symbol vector with elements indexed on c. The user groups $\{u_m\}$ are assigned to subfields in **51** such that subfield c_{M-1} can support 2 users $c_{M-1}=0$ and $c_{M-1}=1$ with each assigned N/2 code symbols c=0,1,. . .,N/2-1, and c=N/2,N/2+1,. . .,N-1 in the N-code block. This enables users in group u_0 to transmit at the symbol rate $R_s=1/2T$. Subfield $c_{M-2}c_{M-1}$ can support 4 users $c_{M-2}c_{M-1}$=00,01,10,11 which allows the users in this group u_2 to transmit at the symbol rate $R_s=1/4T$. In this parsing the subfield elements are the members of the corresponding user groups and the range of the mapping of the subfield onto the field GF(2^M) is the number of symbols in the user group assigned to this subfield. In **51** the menue is defined for the symbol rate, user group, and parsing subfield. Assignment of the parsed subfields to the data vector code slots c is flexible within the constraint that the network operator must distribute the subfields over the code slots c so that the mapping is 1-to-1 which means it must be both unique and non-overlapping.

This mapping as well as the direct mapping of the multiple data rate users onto the data vector enables the Hybrid Walsh block code to have the same flexibility in accommodating multiple data rate users as the real Walsh multiple block codes and with the added advantages of a fast transform, simultaneous transmission of the user data symbols, and the flexibility for assignment of users to contiguous sequency subbands. Examples 1 and 2 in **52** and **53** illustrate representative user assignments to the data fields of the data symbol vector.

Mapping of data input into the data symbol vector **(7)**

49 Data field mapping of data inputs into the data vector

$$\{ Z(u_{m},k_{m}) \} \longrightarrow Z(c)$$

is a linear transformation " $\rightarrow$ " implemented as a multiplexing of the stored input data onto the subfields of c and storing of this data in Z(c) over the N-chip Hybrid Walsh code block and where:

$\{ Z(u_{m},k_{m})$ = input data symbols consisting of user groups $\{u_m\}$ data symbols $\{k_m\}$ over the N-chip CDMA code block defined in **3** in equations **(1)**

Z(c) = data symbol vector for multiple data rate Hybrid Walsh CDMA encoding over N-chip block

50 Mapping of data fields onto data symbol vector Z(c)

Binary representation of code index c

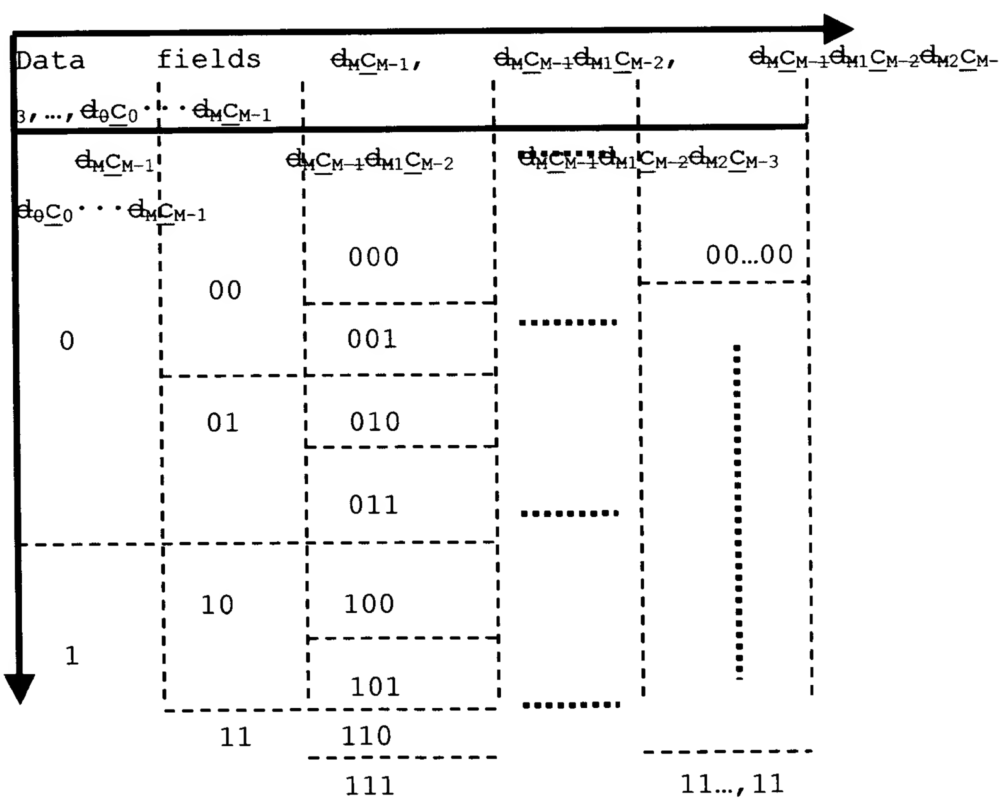
$c = c_0 + c_1 2 + c_2 4 \ . \ . \ . + c_{M-2} N/4 + c_{M-1} N/2$

where $c_0=0,1$, $c_1=0,1$, $c_2=0,1$ . . . are the binary coefficients of c

Digital word

$c = c_0 c_1 c_2 \ . \ . \ . \ c_{M-2} c_{M-1}$

M data fields $d_M c_{M-1}$, $d_M c_{M-1} d_{M2} c_{M-1}$, $d_M c_{M-1} d_{M3} c_{M-2} d_{M2} c_{M-1}$, …, $d_0 c_0 \bullet\bullet\bullet d_M c_{M-1}$



Data symbol vector Z(c) indices c=0,1,. . . ,N-1 ~~N data symbol slots~~

51 Menu of user ~~assignments to the data vector fields~~ assignment to data subfields

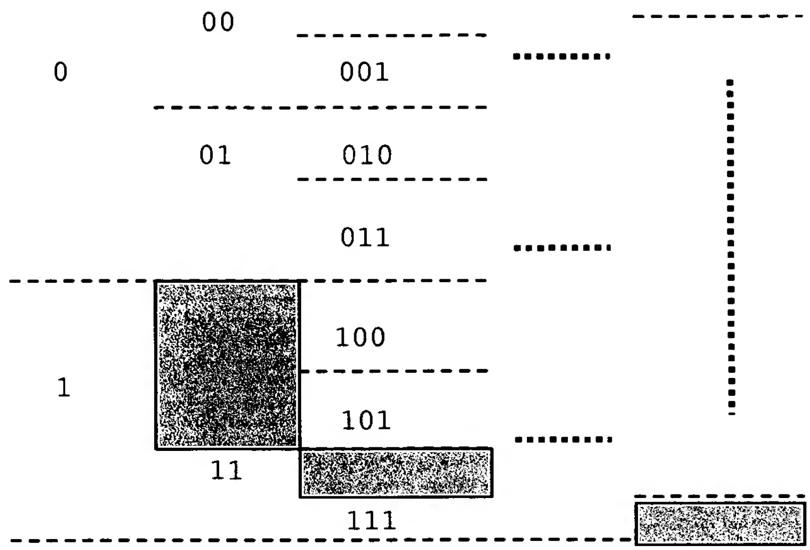
Symbol Rate R_s Symbols/sec.	Users ~~group~~	Data input subfield	
~~u_0~~ 1/2T	$u_0=0$	~~d_M~~ C_{M-1}	=0
	~~u_0~~ =1		=1
~~u_1~~ 1/4T	$u_1=0$	~~d_M~~ C_{M-1} ~~d_{M1}~~ C_{M-2}	=00
	~~u_1~~ =1		=01
	~~u_1~~ =2		=10
	~~u_1~~ =3		=11
⋮	⋮	⋮	
~~u_{M-1}~~ 1/NT	$u_{M-1}=0$	~~d_0~~ $C_0\cdots$ ~~d_M~~ C_{M-1}	$=00\cdots00$
	⋮	⋮	
	~~u_{M-1}~~ =N-1		$=11\cdots11$

~~39~~ **52** Example 1 of multiple data rate menu:

There is 1 user for each group $u_0, u_1, \ldots, u_{M-2}$ and 2 users for u_{M-1} with each user selecting the lowest sequency channel corresponding to the lowest index of channels available to the group.

Example 1 of multiple data rate menu

~~d_M~~ C_{M-1}	~~d_M~~ C_{M-1} ~~d_{M1}~~ C_{M-2}		~~$d_M C_{M-1} d_{M1}$~~ C_{M-2} ~~d_{M2}~~ C_{M-3}	
		000		00...00

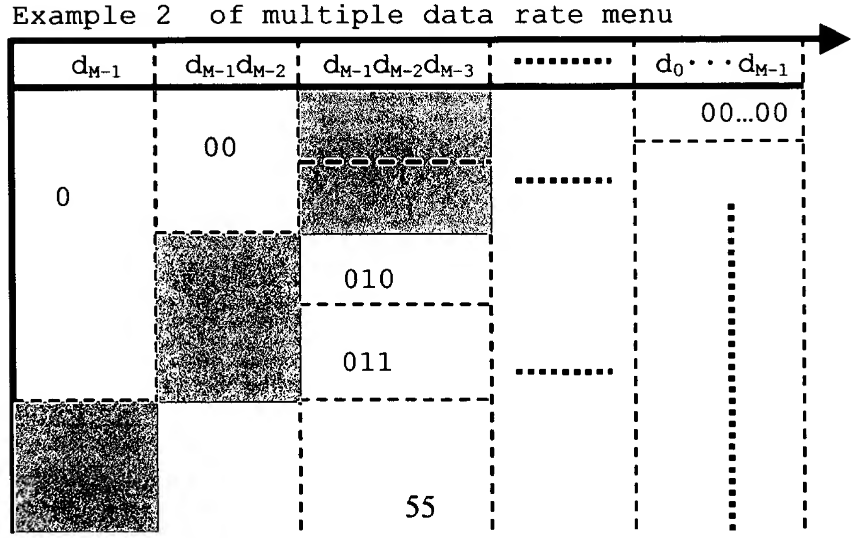


N data symbol vector Z(c) indices c=0,1,. . .,N-1~~N data symbol slots~~

~~40~~ 53 Example 2 of multiple data rate menu:

There is 1 user in each group u_0 and $u_1$and 2 users in u_2 with each user selecting the highest sequency channel corresponding to the highest index of channels available to the group.

Example 2 of multiple data rate menu



```
                10       100
                        -----------
         1
                         101
            -------------------  .........
                 11     110
                        -----------          ----------
                         111                 11,…,11
     -------------------------------------------------
```

N data symbol vector Z(c) indices c=0,1,. . .,N-1~~ N data symbol slots~~

FIG. **2B** depicts a representative Tx encoder implementation for the multiple data rate Hybrid Walsh CDMA encoding algorithms in **45,46** in equations **(6)** using the the data field encoding algorithms in **49-53** in equations **(7)** as well as the direct mapping algorithms and for application to FIG.**1E** for the cellular network and to **1A** for general application upon replacing the real Walsh in **13** by the Hybrid Walsh. This encoder maps the received data symbols $\{Z(u_{m},k_{m})$ for the users $\{u_m\}$ onto the data vector Z(c) which is then CDMA encoded over an N-chip block. Input data symbols for each of the users in groups $u_0, u_1, . . ., u_m, . . ., u_{M-1}$ are received **401** and stored **402** in the respective memories $M_1, M_2, . . . , M_m, . . . , M_{M-1}$. The data from these memories is readout and multiplexed (Mux) **403** onto a single stream of formatted data symbols which define the data symbol vector Z(c) and are stored in memory (Mem) **404**. Mux operation is under control of the Mux algoritms **403** which are either the data field algorithms defined in in **45,46** in equations **(6)** and in **49-53** in equations **(7)**, or are the direct mapping (formatting) algorithms. The arrows in **403** are "1-to-1" and "onto" mappings. Data symbol vector **405** is readout from Mem and multiplied by the Hybrid Walsh code matrix **406** to generate the Hybrid Walsh encoded vector $Z_n(n)$ **407** which is then covered (encoded) by the long and short PN codes **408** to generate the CDMA encoded chip vector **409.**

upon replacing the real Walsh in **27** by the Hybrid Walsh. Inputs **410** are the Rx estimates $\hat{Z}(n)$ of the Tx CDMA encoded chip vectors Z(n). Long and short PN scrambling codes is are removed **411** from $\hat{Z}(n)$ by implementing changing the sign of each chip according to the numerical sign of the real and imaginary components of the complex conjugate of the PN code as per the PN de-covering algorithms **47,48** in equations **(6)**. Next the Hybrid Walsh channelization coding is removed by a matrix multiply operation **412** of the de-covered $\hat{Z}(n)$ with the conjugate transpose $\widetilde{W}'$ of the Hybrid Walsh matrix as defined in **47,48** in equations **(7)**. Output is scaled **413** and stored in Mem **414** as the received estimate $\hat{Z}(c)$ of the Tx data symbol vector Z(c). The $\hat{Z}(c)$ is de-multiplexed (de-Mux) by the de-Mux algorithms **416** which are the reversed (inverse) mappings of the data field algorithms and the direct assignment algorithms for the formatting of the data symbol vector. Arrows indicate "1-to-1" and "onto" mappings.

4. Multiple Data Rate Hybrid Walsh Fast Encoder and Fast Decoder

Fast encoder and decoder algorithms are computationally efficient algorithms since the number of arithmetic add and multiply operations per data symbol are linear in M where N=2^M for a NxN Hybrid Walsh code matrix and which is considerably more

efficient than the linear dependency on N for direct calculation algorithms. The multiple data rate Hybrid Walsh fast encoder algorithm in equation **(8)** and the fast decoder algorithm in equation **(9)** in this invention disclosure are fast algorithms since their number of real adds per data symbol is approximately $R_A \approx 2M+2$ and the number of real multiplies per data symbol is $R_M=0$.

Hybrid Walsh fast encoding and decoding implementation algorithms are defined in equations **(8)**,**(9)**. The fast encoding algorithm in equations **(8)** implements the encoding of the data symbol vector Z(c) with an M pass computation of the Hybrid Walsh encoding and a re-ordering pass **54** followed by PN scrambling **55**. Passes 1,2,3,…,M respectively perform the 2,4,8,…,N chip Hybrid Walsh encoding of the data symbol vector successively starting with the 2 chip encoding in pass 1, the 4 chip encoding in passes 1,2, the 8 chip encoding in pass 1,2,3, and the N chip encoding in passes 1,2,3,…,M where $N=2^M$. This fast algorithm is a computationally efficient means to implement the Hybrid Walsh encoding of each N chip code block for multiple data rate users whose lowest data rate corresponds to the data symbol rate of an N chip encoded user. It is easily demonstrated that the number of real additions R_A per data symbol is approximately equal to $RA \approx 2M+2$ in the implementation of this fast algorithm **41**, where $N=2^M$. For the real Walsh encoding it is well known that the fast algorithm requires $R_A \approx M+1$ real additions per data symbol. Although the number of real adds has been doubled in using the Hybrid Walsh compared to the real Walsh, the add operations are a low complexity implementation cost which means that the Hybrid Walsh maintains its attractiveness as a zero-multiplication CDMA encoding orthogonal code set. This fast algorithm generates the complex Walsh CDMA encoded chips in bit reversed order. A re-ordering pass can changes the bit reversed output to the normal output. There are

other variations to this algorithm such as starting with the computation of n_0 and proceeding to pass M to calculate n_{M-1}.

~~41Complex Walsh encoding and channel combining uses a computationally efficient fast encoding algorithm. This algorithm implements the encoding with an M pass computation. Passes 1,2,3,…,M respectively perform the 2,4,8,…,N chip complex Walsh encoding of the data symbol vector successively starting with the 2 chip encoding in pass 1, the 4 chip encoding in passes 1,2, the 8 chip encoding in pass 1,2,3, and the N chip encoding in passes 1,2,3,…,M where N=2^M. Using the binary word representations for both d and n, this M pass algorithm is:~~

Hybrid Walsh CDMA fast encoding **(8)**

for multiple data rate users

54 Hybrid Walsh fast encoding

Pass 1: $Z^{(1)}(n_{M-\cancel{1}}\cancel{d_{11}}c_1\cdots\cancel{d_M}c_{M-1})$

$$= \sum Z(\cancel{d_0}c_0\cdots\cancel{d_M}c_{M-1})\,[(-1)^\wedge\cancel{dr_0 n_M}cr_0 n_{M-1} + j(-1)^\wedge\cancel{di_0 n_M}ci_0 n_{M-1}]$$

$$\cancel{d_0}c_0=\cancel{dr_0}cr_0=\cancel{di_0}ci_0=0,1$$

⋮

Pass m for m=2,…,M-1

$$Z^{(m)}(n_{M-1}\cdots n_{M-\cancel{m}}\cancel{d_{mm}}c_m\cdots\cancel{d_M}c_{M-1})$$

$$= \sum Z^{(m-1)}(n_{M-1}\cdots n_{M-m+\cancel{1}}\cancel{d_{m1}}c_{m-1}\cdots\cancel{d_M}c_{M-1})\cdot$$

$$[(-1)^\wedge\cancel{dr_m}cr_{m-1}(n_{M-m}+n_{M-m+1})+j(-1)^\wedge\cancel{di_m}ci_{m-1}(n_{M-m}+n_{M-m+1}])$$

$$\cancel{d_m}c_{m-1}=\cancel{dr_m}cr_{m-1}=\cancel{di_m}ci_{m-1}=0,1$$

⋮

Pass M: $Z^{(M)}(n_{M-1}n_{M-2}\cdots n_1 n_0)$

$$=\sum Z^{(M-1)}(n_{M-1}n_{M-2}\cdots n_1 d_{M-1})\cdot$$

$$[(-1)^\wedge\cancel{dr_M}cr_{M-1}(n_0+n_1)+j(-1)^\wedge\ \cancel{di_M}ci_{M-1}(n_0+n_1)]$$

$\cancel{d_M}\underline{c}_{M-1}=\cancel{dr_M}\underline{cr}_{M-1}=\cancel{di_M}\underline{ci}_{M-1}=0,1$

$= \cancel{\tilde{\tilde{Z}}}\ \underline{Z}_n(n_{M-1}n_{M-2}\bullet\bullet\bullet n_1 n_0)$

~~An additional re~~Re-ordering pass is added to change The ~~encoded N-chip block~~ $\cancel{\tilde{\tilde{Z}}}\underline{Z}_n(n_{M-1}n_{M-2}\bullet\bullet\bullet n_1 n_0)$ in bit Reversed ordering to the normal readou~~u~~

$$\mathbf{Z(n_0 n_1 \bullet\bullet\bullet n_{M-2} n_{M-1}) = Z(n)}$$

~~**42**~~—PN scrambling

~~$P_R(n)$, $P_I(n)$ = PN code chip n for real and Imaginary axes~~

Z(n) = PN scrambled ~~complex~~ Hybrid Walsh encoded data chips ~~after summing over the users~~

$= \cancel{\sum_u \tilde{\mathbf{Z}}(\mathbf{n})\, P_2(n)[P_R(n) + j P_I(n)]}$

$= \cancel{\sum_u \tilde{\mathbf{Z}}(\mathbf{n})\, \mathrm{sign}\{P_2(n)\}[\mathrm{sign}\{P_R(n)\} + j\,\mathrm{sign}\{P_I(n)\}]}$

$= Z_n(n)P_2(n)[P_R(n)+j\ P_I(n)]$

$= Z_n(n)\,\mathrm{sign}\{P_2(n)\}\ [\mathrm{sign}\{P_R(n)\} +j\ \mathrm{sign}\{P_I(n)\}]$

The fast decoding algorithm in equations **(9)** implements the decoding of the Rx CDMA encoded chip vector $\hat{Z}(n)$ starting with the removal of the PN scrambling **56** to yield the Hybrid Walsh encoded chip vector $\hat{Z}_n(n)$ and followed by an M-pass computation of the Hybrid Walsh decoding and a re-ordering plus a rescaling pass to yield the Rx estimate of the transmitted data symbol vector $\hat{Z}(c)$. Passes 1,2,3,…,M respectively perform the 2,4,8,…,N chip Hybrid Walsh decoding of the encoded chip vector successively starting with the 2 chip decoding in pass 1, the 4 chip decoding in passes 1,2, the 8 chip decoding in pass 1,2,3,

and the N chip decoding in passes 1,2,3,…,M where $N=2^M$. Like the fast encoding algorithm in equations **(8)** this fast decoding algorithm in equations **(9)** is a computationally efficient means to implement the Hybrid Walsh decoding of each N-chip encoded vector for multiple data rate users whose lowest data rate corresponds to the data symbol rate of an N-chip encoded user. Identical to the encoding complexity, it is easily demonstrated that the number of real additions R_A per data symbol is approximately equal to $R_A \approx 2M+2$ in the implementation of this fast algorithm where $N=2^M$. For the real Walsh decoding it is well known that the fast algorithm requires $R_A \approx M+1$ real additions per data symbol. Similar to the observations on the fast encoding, the fast decoding supports the Hybrid Walsh as a zero-multiplication computationally efficient CDMA decoding orthogonal code set. A re-ordering pass changes the bit reversed output to the normal output. There are other variations to this algorithm such as starting with the computation of n_{M-1} and proceeding to pass M to calculate n_0.

Hybrid Walsh CDMA fast decoding **(9)**

56 PN scrambling

$\hat{Z}_n(n)$ = PN descrambled CDMA encoded chips $\hat{Z}(n)$

$= \hat{Z}(n) P_2(n)\,[P_R(n)+j\ P_I(n)]$

$= \hat{Z}(n)\ \mathrm{sign}\{P_2(n)\}\,[\mathrm{sign}\{P_R(n)\}+j\ \mathrm{sign}\{P_I(n)\}]$

57 Hybrid Walsh fast decoding

Pass 1: $Z^{(1)}(c_{M-1} d_{11} n_1 \cdots d_M n_{M-1})$

$= \Sigma\ \hat{Z}_n(d_0 n_0 \cdots d_M n_{M-1})\,[(-1)\text{^} dr_0 n_M n_0\ \ cr_{M-1}\ +\ j(-1)\text{^}\ n_0$ $di_0 n_M ci_{M-1}]$

$d_0 n_0 = dr_0 0,1$

Pass m for m=2,…,M-1

$$Z^{(m)}(c_{M-1}\cdots c_{M-m} d_{mm} n_m \cdots d_M n_{M-1})$$

$$= \sum Z^{(m-1)}(c_{M-1}\cdots c_{M-m+1} d_{m1} n_{m-1}\cdots d_M n_{M-1})\cdot$$

$$[\ (-1)\hat{\ }n_{m-1}(cr_{M-m+1} + cr_{M-m})$$

$$+ j(-1)\hat{\ }n_{m-1}(ci_{M-m+1} + ci_{M-m})$$

$$d_m n_{m-1} = dr_m 0,1$$

⋮

Pass M: $Z^{(M)}(c_{M-1}c_{M-2}\cdots c_1 c_0)$

$$= \sum Z^{(M-1)}(c_{M-1}c_{M-2}\cdots c_1 n_{M-1})\cdot$$

$$[\ (-1)\hat{\ }n_{M-1}(cr_0 + cr_1)$$

$$+ j(-1)\hat{\ }n_{M-1}(ci_0 + ci_1)$$

$$d_M n_{M-1} = dr_M 0,1$$

$$= \hat{Z}(c_{M-1}c_{M-2}\bullet\bullet\bullet c_1 c_0)$$

Reordering and rescaling pass:

$$\hat{Z}(c) = \hat{Z}(c_0 c_1 \bullet\bullet\bullet c_{M-2} c_{M-1})$$

$$= (1/4N)\ f[\hat{Z}(c_{M-1}c_{M-2}\bullet\bullet\bullet c_1 c_0)]$$

$$= (1/4N)\ f[Z^{(M)}(c_{M-1}c_{M-2}\bullet\bullet\bullet c_1 c_0)]$$

where $f[\hat{Z}], f[Z^{(M)}]$ is the bit reversed value of $\hat{Z}$, $Z^{(M}$

FIG. **5B** depicts a representative implementation block diagram for the Tx fast encoder algorithm in equations **(8)** for multiple data rate Hybrid Walsh CDMA encoding and executes the fast encoder algorithm in the encoder implementation in FIG. **2B.** Received data symbols **418** are mapped by the Mux algorithm **420** into the data symbol vector V(c) memory Mem **419.** The data symbol vector V(c) is processed by the Hybrid Walsh fast encoding algorithm in equations **(8)** by executing M passes **421** starting with pass 1 whose output is the partially processed data vector $Z^{(1)}$ and continuing through pass M with output Z^M which is reordered in another pass and handed over to the Hybrid Walsh encoded vector $Z_n(n)$ memory Mem **422.** This vector **423** is

scrambled by the long and short PN codes **424** to generate the CDMA encoded chip vector Z(n) **425.**

FIG. **6B** depicts a representative implementation block diagram for the Rx fast decoder algorithm in equations **(9)** for multiple data rate Hybrid Walsh CDMA decoding and executes the fast decoder algorithm in the decoder implementation in FIG. **4B.** Inputs **426** are the Rx estimates $\hat{Z}(n)$ of the Tx CDMA encoded chip vectors Z(n). Long and short PN scrambling codes is are removed **427** from $\hat{Z}(n)$ to yield the Rx estimate $\hat{Z}_n(n)$ **428** of the Tx Hybrid Walsh encoded chips $Z_n(n)$. The $Z_n(n)$ is processed by the Hybrid Walsh fast decoding algorithm in equations **(8)** by executing M passes **429** starting with pass 1 whose output is the partially processed data vector $Z^{(1)}$ and continuing through pass M with output Z^M which is reordered in another pass and rescaled by multiplying by the factor 1/2N and handed over to the data symbol vector $\hat{Z}(c)$ memory Mem **430.** The $\hat{Z}(c)$ is de-Multiplexed **431** by the de-Mux algorithms **432** which are the reversed (inverse) mappings of the data field algorithms and the direct assignment algorithms for the formatting of the data symbol vector, to output the Rx estimates $\{\hat{Z}(u_{m,k_m})\}$ **433** of the Tx user complex data symbols $\{Z(u_{m,k_m})\}$.

~~The fast algorithm in **41** is a computationally efficient means to implement the complex Walsh encoding of each N chip code block for multiple data rate users whose lowest data rate corresponds to the data symbol rate of an N chip encoded user. It is easily demonstrated that the number of real additions~~ ~~R_A per data symbol is approximately equal to RA≈2M+2 in the implementation of this fast algorithm **41**, where $N=2^M$. For the real Walsh encoding it is well known that the fast algorithm requires R_A≈M+1 real additions per data symbol. Although the number of real adds has been doubled in using the complex Walsh~~

~~compared to the real Walsh, the add operations are a low complexity implementation cost which means that the complex Walsh maintains its attractiveness as a zero-multiplication CDMA encoding orthogonal code set.~~

~~The fast algorithm in **41** consists of M signal processing passes on the stored data symbols to generate the complex Walsh CDMA encoded chips in bit reversed order. A re-ordering pass can changes the bit-reversed output to the normal output. Advantage is taken of the equality c=d which allows the d to be used in the code indices for the complex Walsh: $d_m=c_m$, dr=cr, di=ci. Pass 1 implements 2 chip encoding, pass 2 implements 4 chip encoding, ….,and the last pass M performs $N=2^M$ chip encoding.~~

~~PN scrambling of the complex Walsh CDMA encoded chips in **42** is accomplished by encoding the $\{\tilde{Z}(n)\}$ with a complex PN which is constructed as the complex code sequence $[P_R(n)+jP_I(n)]$ wherein $P_R(n)$ and $P_I(n)$ are independent PN sequences used for the real and imaginary axes of the complex PN. These PN codes are 2-phase with each chip equal to +/-1 which means PN encoding consists of sign changes with each sign change corresponding to the sign of the PN chip. Encoding with PN means each chip of the summed complex Walsh encoded data symbols has a sign change when the corresponding PN chip is -1, and remains unchanged for +1 values. This operation is described by a multiplication of each chip of the summed complex Walsh encoded data symbols with the sign of the PN chip. Purpose of the PN encoding for complex data symbols is to provide scrambling of the summed complex Walsh encoded data symbols as well as isolation between groups of users. Output of this complex Walsh CDMA encoding followed by the complex PN scrambling are the CDMA encoded chips over the N chip block $\{Z(n)\}$.~~

5. Multiple Data Rate Generalized Hybrid Walsh Fast Encoder and Fast Decoder

Fast encoder and decoder algorithms are computationally efficient algorithms since their arithmetic add and multiply operations per data symbol are linear in the $\{M_n\}$ where $N_n=2^{\wedge}M_n$ is the size of one of the code matrices indexed on "n" in the construction of the generalized Hybrid Walsh code matrix and which is considerably more efficient than the linear dependency on $N=N_0.\ .\ .N_n.\ .\ .$ for direct calculation algorithms. The multiple data rate Hybrid Walsh fast encoder algorithm in equation **(10)** and the fast decoder algorithm in equation **(11)** in this invention disclosure are fast algorithms since their number of real adds per data symbol is approximately $R_A \approx 2M+M_1+2$ and the number of real multiplies per data symbol is $R_M=2M_1$ where M refers to tensor product of the Hybrid Walsh code matrix and M_1 refers to the DFT code matrix.

Equations **(10)** define the fast algorithm for the Tx encoding of the multiple data rate generalized Hybrid Walsh CDMA orthogonal codes for the representative example **58** which constructs the NxN generalized Hybrid Walsh orthogonal CDMA code matrix $C_N= \widetilde{W}_{N_0} \otimes E_{N_1}$ as the tensor product of the $N_0 x N_0$ Hybrid Walsh $\widetilde{W}_{N_0}$ and the $N_1 x N_1$ complex DFT E_{N_1}, where $N=N_0 N_1$. Chip element equations are $C_N(c,n)= \widetilde{W}_{N_0}(c\widetilde{w},n\widetilde{w})\, E_{N_1}(ce,ne)$ with $c = ce + c\widetilde{w} N_1$ and $n = ne + n\widetilde{w} N_1$ since C_N is the generalized Hybrid Walsh code matrix with each element of $\widetilde{W}_{N_0}$ replaced the matrix E_{N_1}. The binary representation of c,n in **59** is used in the Mux algorithms that map the multiple data rate user symbols into the data symbol vector Z(c) and also are used in the development of the fast encoding algorithm. These c,n binary representations differ from those in **45** in equations **(6)** in the

inclusion of the tensor product in the definitions of c,n. Fast encoding **60** encodes the data symbol vector Z(c) with an $M=M_0M_1$-pass computation starting with the M_0-pass computation of the Hybrid Walsh encoding and followed by the M_1-pass computation of the DFT encoding and followed by a re-ordering pass and then by the long and short code PN scrambling **61**. Similar to the Hybrid Walsh fast encoding algorithm in **54** in equations **(8)**, passes $1,2,3,\ldots,M_0$ respectively perform the $2,4,8,\ldots,N_0$ chip Hybrid Walsh encoding of the data symbol vector and passes $M_0+1,\ .\ .\ .\ ,M=M_0+M_1$ respectively perform the $2,4,8,\ldots,N_1$ chip DFT encoding. This fast algorithm generates the generalized Hybrid Walsh CDMA encoded chips in bit reversed order. A re-ordering pass changes the bit reversed output to the normal output $Z_n(n)$ which is scrambled by the PN codes **61** to yield the CDMA encoded chip vector Z(n). There are other variations to this algorithm such as starting with the computation of n_0 and proceeding to pass M to calculate n_{M-1}.

This fast algorithm is a computationally efficient means to implement the generalized Hybrid Walsh encoding of each N-chip code block for multiple data rate users whose lowest data rate corresponds to the data symbol rate of an N-chip encoded user. It is easily demonstrated that the number of real additions R_A per data symbol is approximately equal to $R_A \approx 2M+M_1+2$ and the number of real multiplies R_M per data symbol is $R_M \approx 2M_1$ in the implementation of this fast algorithm where $N=2^M$. Inclusion of the DFT in the generalized Hybrid Walsh adds some multiplies to the computational complexity with the benefit of increasing the choices for the code length N.

The mathematical definition and the implementation of the fast encoding algorithm for this example of the generalized Hybrid Walsh are sufficiently detailed to enable this algorithm and implementation to be applied to generalized Hybrid Walsh CDMA

codes by someone skilled in the art of CDMA communications and fast transforms.

~~**Transmitter equations (6) for hybrid complex Walsh orthogonal encoding** of multiple data rate users are derived by starting with the hybrid complex Walsh orthogonal codes disclosed in the invention application [6]. The discrete Fourier transform (DFT) CDMA codes used in the example generation of hybrid complex Walsh orthogonal CDMA codes in [6] are given in equations (4) along with a fast encoding algorithm.~~

~~**N-chip DFT complex orthogonal CDMA codes** (4)~~

~~**43** DFT code vectors~~

~~E_N = DFT NxN orthogonal code matrix consisting of N rows of N chip code vectors~~

~~= [$E_N(c)$] matrix of row vectors $E(c)$~~

~~= [$E_N(c,n)$] matrix of elements $E(c,n)$~~

~~$E_N(c)$ = DFT code vector c~~

~~= [$E_N(c,0)$, $E_N(c,1)$, ..., $E_N(c,N-1)$]~~

~~= 1xN row vector of chips $E_N(c,0)$,...., $E_N(c,N-1)$~~

~~$E_N(c,n)$ = DFT code c chip n~~

~~$= e^{j2\pi\, cn/N}$~~

~~$= \cos(2\pi\, cn/N) + j\sin(2\pi\, cn/N)$~~

~~= N possible values on the unit circle~~

~~**44** Fast encoding algorithm for N chip block of data in the data vector $d=d_0d_1\cdots d_{M-2}\,d_{M-1}$~~

~~Pass 1: $Z^{(1)}(d_0d_1\cdots d_{M-2}\,n_0)$~~

~~$= \sum_{d_{M-1}=0,1} Z(d_0d_1d_2\cdots d_{M-2}d_{M-1})\,(e^{j2\pi})^{\wedge}d_{M-1}n_0/2$~~

$\vdots$

~~Pass m for m=2,...,M-1~~

~~$Z^{(m)}(d_0 \cdots d_{M-m-1} n_{m-1} \cdots n_0)$~~

~~$= \sum_{d_{M-m}} Z^{(m-1)}(d_0 \cdots d_{M-m} n_{m-2} \cdots n_0) \bullet e\hat{\ }j2\pi[d_{M-m}(n_0+n_1 2+\cdots+n_{m-1}2\hat{\ }(m-1))/2^m]$~~

⋮

~~Pass M for m=2,…,M-1~~

~~$Z^{(M)}(n_{M-1} \cdots n_0)$~~

~~$= \sum_{d_0} Z^{(m-1)}(d_0 n_{M-2} \cdots n_0) \bullet e\hat{\ }j2\pi[d_0(n_0+n_1 2+\cdots+n_{M-1}2\hat{\ }(M-1))/2^M]$~~

~~$= \tilde{Z}(n_{M-1}n_{M-2} \cdots n_1 n_0)$~~

~~An additional re-ordering pass is added to change the encoded N chip block $\tilde{Z}(n_{M-1}n_{M-2}\bullet\bullet\bullet n_1 n_0)$ in bit reversed ordering to the normal readout ordering~~

~~$\tilde{Z}(n_0 n_1 \bullet\bullet\bullet n_{M-2} n_{M-1}) = \tilde{Z}(n)$~~

~~An additional re-ordering pass is added to change the encoded N chip block $\tilde{Z}(n_{M-1}n_{M-2}\bullet\bullet\bullet n_1 n_0)$ in bit reversed ordering to the normal readout ordering~~

~~$\tilde{Z}(n_0 n_1 \bullet\bullet\bullet n_{M-2} n_{M-1}) = \tilde{Z}(n)$~~

~~DFT code matrix and the row code vectors are defined in~~ **43** ~~for an N chip block. A fast algorithm for the encoding of the N chip data vector $Z(d_0 d_1 d_2 \cdots d_{M-2} d_{M-1})$ is defined in~~ **44** ~~in a format similar to the fast algorithm for the complex Walsh encoding in equations (3). It is well known that the computational complexity of the fast DFT encoding algorithm is $R_A \approx 2M$ real additions per data symbol plus $R_M \approx 3M$ real multiplications per data symbol. The relatively high complexity implementation cost of multiplies makes it desirable to limit the use of DFT codes to~~

~~applications such as the hybrid complex Walsh wherein the number of real multiplies per data symbol can be kept more reasonable.~~

~~A fast algorithm for the encoding of the hybrid complex Walsh CDMA orthogonal codes is described in equations **(6)** for the representative example **48** which constructs the NxN hybrid complex Walsh orthogonal CDMA code matrix $C_N = \tilde{W}_{N_0} \otimes E_{N_1}$ as the Kronecker product of the $N_0 x N_0$ complex Walsh $\tilde{\mathbf{W}}_{N_0}$ and the $N_1 x N_1$ complex DFT, where $N=N_0N_1$. Each chip element of C_N is the product **49** of the chip elements of the complex Walsh and complex DFT code matrices. The complex Walsh and DFT codes are phase codes which means the phase of each C_N chip element is the sum of the phases of the chip elements for the complex Walsh and complex DFT. Chip element equations are $C_N(c,n) = \tilde{W}_{N_0}(c\tilde{w}, n\tilde{w}) E_{N_1}(ce, ne)$ with $c = ce + c\tilde{w}N_1$ and $n = ne + n\tilde{w}N_1$. For multiple data rate data symbol assignments and for the construction of the fast encoding algorithm, it is convenient to use a binary word representation of the chip element indices c,n. Binary word representation **50** is $c = ce_0 ce_1 \cdots ce_{M_1-1} c\tilde{w}_{M_1} c\tilde{w}_{M_1+1} \cdots c\tilde{w}_{M-1} = c_0 c_1 \cdots c_{M-2} c_{M-1}$ where the first binary word is a function of the binary words for the complex Walsh and complex DFT code indices, and the second binary word is a direct representation of the C_N indices which will be used for the data vector construction. The same binary word representations apply for the chip index n upon substituting the n for c. User data **38** in equations (3) for the N chip code block is mapped into the N data symbol vector $d = d_0 \cdots d_{M-1}$ which is obtained from the binary word for c by substituting the index d for the index c in the binary word representation.~~

~~The multiple data rate data symbol mapping **51** in equations (6) for the hybrid complex Walsh codes remains the same as used~~

~~in **38, 39, 40** in equations (3) for the complex Walsh codes. The data symbol mapping assigns the N/2 data symbols of the 2 chip data symbol transmission rate users to the d_{M-1} field, the N/4 data symbols of the 4 chip data symbol transmission rate users are assigned to the $d_{M-1}d_{M-2}$ field, ..., and the single data symbols of the N chip data symbol transmission rate users are assigned to the $d_0 \cdots d_{M-1}$ field, where the data vector index "d" is represented as the binary number $d=d_0 \cdots d_{M-1}$ and the $\{d_m\}$ are the binary coefficients. For a fully loaded CDMA communications frequency band the N data symbols occupy the N available data symbol locations in the data symbol vector $d= d_0 \cdots d_{M-1}$. The menu of available user assignments to the data vector fields is given in **38** in equations (3). Examples 1 and 2 in **39** and **40** in equations (3) illustrate representative user assignments to the data fields of the data symbol vector. This mapping of the user data symbols into the data symbol vector is equivalent to setting c=d which makes it possible to develop the fast encoding algorithm **51**.~~

Generalized Hybrid Walsh fast encoding **(10)**

for multiple data rate users

58 Example NxN generalized Hybrid Walsh code matrix C_N

$C_N = \widetilde{W}_{N_0} \otimes E_{N_1}$ tensor product of $\widetilde{W}_{N_0}$ and E_{N_1}

$= [C_N(c)]$ matrix of row vectors $C_N(c)$

$= [C_N(c,n)]$ matrix of elements $C_N(c,n)$ ~~**Fast multiple data rate hybrid complex Walsh encoding (6)**~~

~~**for transmitter**~~

~~**48** The fast algorithm will be described for the example NxN complex orthogonal CDMA code matrix C_N which is generated by the Kronecker product of the N_0xN_0 complex Walsh matrix $\widetilde{W}_{N_0}$ and the complex N_1xN_1 DFT matrix E_{N_1}~~

~~C_N = Kronecker product of $\widetilde{W}_{N_0}$ and E_{N_1}~~

~~$= \widetilde{W}_{N_0} \otimes E_{N_1}$~~

where $N = N_0 N_1$

$= 2\text{^}M$

$M = M_0 + M_1$

$N_0 = 2\text{^}M_0$

$N_1 = 2\text{^}N_1$

~~**49** N chip hybrid complex Walsh code block C_N~~

~~C_N = hybrid complex Walsh NxN orthogonal code matrix consisting of N rows of N chip code vectors~~

~~= $[C_N(c)]$ matrix of row vectors $C_N(c)$~~

~~= $[C_N(c,n)]$ matrix of elements $C_N(c,n)$~~

$C_N(c,n)$ = ~~hybrid complex Walsh code c chip n~~

$$= \widetilde{W}_{N_0}(c\tilde{w}, n\tilde{w})\, E_{N_1}(ce, ne)$$

~~$= [+/-1 \; +/-j] \; E_{N_1}(ce,ne)$ values~~

where $c = ce + c\tilde{w} N_1$

$n = ne + n\tilde{w} N_1$

~~**50**~~ **59** Binary representation of c,n ~~indexing of codes~~ in the matrix C_N

$$\begin{aligned}
\mathbf{c} &= \mathbf{ce}_0 + \mathrm{ce}_1 2 + \cdots + \mathrm{ce}_{\mathrm{M}_1\text{-}1} 2\text{^}(\mathbf{M}_1 - 1) \\
&\quad + \mathrm{c}\tilde{\mathrm{w}}_{\mathrm{M}_1} 2\text{^}\mathbf{M}_1 + \mathrm{c}\tilde{\mathrm{w}}_{\mathrm{M}_1+1} 2\text{^}(\mathbf{M}_1 + 1) + \cdots + \mathbf{c}\tilde{\mathbf{w}}_{\mathbf{M}-1} 2\text{^}\mathbf{M} - 1 \\
&= \mathrm{ce}_0 \mathbf{ce}_1 \cdots \mathbf{ce}_{\mathrm{M}_1-1} \mathbf{c}\tilde{\mathbf{w}}_{\mathrm{M}_1} \mathbf{c}\tilde{\mathbf{w}}_{\mathrm{M}_1+1} \cdots \mathbf{c}\tilde{\mathbf{w}}_{\mathbf{M}-1} \qquad \text{Binary word} \\
&= c_0 c_1 \cdots\cdots c_{M-1} \qquad \text{Binary word} \\
\mathbf{n} &= \mathbf{ne}_0 + \mathrm{ne}_1 2 + \cdots + \mathrm{ne}_{\mathrm{M}_1\text{-}1} 2\text{^}(\mathbf{M}_1 - 1) \\
&\quad + \mathrm{n}\tilde{\mathrm{w}}_{\mathrm{M}_1} 2\text{^}\mathbf{M}_1 + \mathrm{n}\tilde{\mathrm{w}}_{\mathrm{M}_1+1} 2\text{^}(\mathbf{M}_1 + 1) + \cdots + \mathbf{n}\tilde{\mathbf{w}}_{\mathbf{M}-1} 2\text{^}\mathbf{M} - 1 \\
&= \mathrm{ne}_0 \mathbf{ne}_1 \cdots \mathbf{ne}_{\mathrm{M}_1-1} \mathbf{n}\tilde{\mathbf{w}}_{\mathrm{M}_1} \mathbf{n}\tilde{\mathbf{w}}_{\mathrm{M}_1+1} \cdots \mathbf{n}\tilde{\mathbf{w}}_{\mathbf{M}-1} \qquad \text{Binary word}
\end{aligned}$$

~~51~~1. ~~The fast encoding algorithm starts with the data symbol vector d and mapping of the user groups u_0, u_1, ..., u_{M-1} into the data fields of d. This mapping is identical to the mapping defined in equations (3) for the multiple data rate complex Walsh orthogonal encoding of the CDMA over an N chip block. However,~~

~~the fast algorithm for the hybrid complex Walsh encoding is modified to accommodate the Kronecker construction as illustrated by the following fast algorithm for the hybrid complex Walsh example in **48**. Using the binary representations of d,n~~

$$\cancel{d = d_0 d_1 \cdots\cdots d_{M_1-1} d_{M_1} \cdots\cdots d_{M-1}}$$
$$\cancel{= de_0 de_1 \cdots de_{M_1-1} d\tilde{w}_{M_1} \cdots d\tilde{w}_{M-1}}$$
$$\cancel{n = n_0 n_1 \cdots\cdots n_{M_1-1} n_{M_1} \cdots\cdots n_{M-1}}$$
$$\cancel{= ne_0 ne_1 \cdots ne_{M_1-1} n\tilde{w}_{M_1} \cdots n\tilde{w}_{M-1}}$$

~~and the same approach used to derive the fast algorithms **41** in equations (3) and **44** in equations (4), enables the M-pass fast algorithm to be defined~~

60 Fast encoding of generalized Hybrid Walsh

Pass 1 for ~~complex~~ Hybrid Walsh codes

$$Z^{(1)}(c_0 \cdots c_{M_1-1}\; n_{M_0-1}\; c_{M_1+1} \cdots c_{M-1})$$
$$= \sum_{c_{M_1} = cr_0 = ci_0 = 0,1} Z(c_0 \cdots c_{M-1}) \cdot [(-1)^{\wedge} cr_0\; n_{M_0-1} + j(-1)^{\wedge}\; ci_0\; n_{M_0-1}]$$

where the Hybrid Walsh indexing reduces to

$$cr_0 = cr_{M_1} \mod(M_1)$$
$$ci_0 = ci_{M_1} \mod(M_1)$$

$$\cancel{Z^{(1)}(d_0 \cdots d_{M_1-1}\, n_{M1}\, d_{M_1+1} \cdots d_{M-1})}$$
$$\cancel{= \sum Z(d_0 \cdots d_{M-1}) \cdot [(-1)^{\wedge} dr_{M1}\, n_{M_1} + j(-1)^{\wedge}\, di_{M1}\, n_{M_1} +]}$$

$$\cancel{d_{M_1} = dr_{M_1} = di_{M_1} = 0,1}$$

Pass m for m=2,…,M_0-1 for ~~complex~~ Hybrid Walsh codes

$$Z^{(m)}(c_0 \cdots c_{M_1-1} n_{M_0-1} \cdots n_{M_0-m}\; c_{M_1+m} \cdots c_{M-1})$$

$$= \sum Z^{(m-1)}(\, c_0 \cdots c_{M_1-1} n_{M_0-1} \cdots n_{M_0-m+1}\; c_{M_1+m-1} \cdots c_{M-1}) \cdot$$

$$[(-1)\wedge cr_{m-1}\,(n_{M_0-m} + n_{M_0-m+1}) \pm$$

$$+j(-1)\wedge ci_{m-1}\,(n_{M_0-m} + n_{M_0-m+1})]$$

$$c_{M_1+m-1} = cr_{m-1} = ci_{m-1} = 0,1$$

where the Hybrid Walsh indexing reduces to

$$cr_{m-1} = cr_{M_1+m-1} \bmod(M_1)$$

$$ci_{m-1} = ci_{M_1+m-1} \bmod(M_1)$$

$$\cancel{Z^{(m)}(d_0 \cdots d_{M_1-1} n_{M_0-1} \cdots n_{M_0-m}\, d_{M_1+m} \cdots d_{M-1})}$$

$$\cancel{= \sum Z^{(m-1)}(\, d_0 \cdots d_{M_1-1} n_{M_0-1} \cdots n_{M_0-m+1}\, d_{M_1+m-1} \cdots d_{M-1}) \cdot}$$

$$\cancel{[(-1)\wedge dr_{m-1}(n_{M_0-m} + n_{M_0-m+1}) +}$$

$$\cancel{+j(-1)\wedge di_{m-1}(n_{M_0-m} + n_{M_0-m+1})]}$$

$$\cancel{d_{M_1+m-1} = dr_{m-1} = di_{m-1} = 0,1}$$

⋮

Pass M_0 for Hybrid Walsh codes

$$Z^{(M_0)}(c_0 \cdots c_{M_1-1} n_{M_0-1} \cdots n_0)$$

$$= \Sigma \; Z^{(M-1)}(c_0 \cdots c_{M_1-1} n_{M_0-1} \cdots n_1 c_{M-1}) \cdot$$

$$[(-1)^\wedge cr_{M_0-1}(n_0+n_1)$$

$$+j(-1)^\wedge ci_{M_0-1}(n_0+n_1)]$$

$$c_{M-1} = cr_{M_0-1} = ci_{M_0-1} = 0,1$$

where the Hybrid Walsh indexing reduces to

$$cr_{M_0-1} = cr_{M-1} \bmod (M_1)$$

$$ci_{M_0-1} = ci_{M-1} \bmod (M_1)$$

⋮

Pass $M_0+m=M_0+1, \ldots, M_0+M_1-1=M-1$ for ~~complex~~ DFT codes

$$Z^{(M_0+m)}(c_0 \cdots c_{M_1-m-1} \; n_{M_0+m-1} \cdots n_0)$$

$$= \Sigma Z^{(M_0+m-1)}(c_0 \cdots c_{M_1-m} \; n_{M_0+m-2} \cdots n_0) \bullet$$

$$[e^\wedge(-)j2\pi \; c_{M_1-m}(n_{M_0} + n_{M_0+1}2 + \cdots + n_{M_0+m-1}2^\wedge m-1)/2^\wedge m]$$

$$c_{M_1-m} = 0,1$$

$$\cancel{Z^{(M_0+m)}(d_0 \cdots d_{M_1-m-1} n_{M_0+m-1} \cdots n_0)}$$

$$\cancel{= \Sigma Z^{(M_0+m-1)}(d_0 \cdots d_{M_1-m} n_{M_0+m-2} \cdots n_0) \bullet}$$

$$\cancel{[e^\wedge j2\pi)^\wedge d_{M_1-m}(n_{M_0} + n_{M_0+1}2 + \cdots + n_{M_0+m-1}2^\wedge m-1)/2^\wedge m]}$$

$$\cancel{d_{M_1-m} = 0,1}$$

⋮

Pass M for ~~complex~~ DFT codes

Pass M for DFT codes

$$Z^{(M)}(n_{M-1}\cdots n_1 n_0)$$

$$= \sum_{c_0=0,1} Z^{(M-1)}(c_0 n_{M-2}\cdots n_1 n_0)\bullet [e^{\wedge}(-)j2\pi c_0(n_{M_0} + n_{M_0+1}2+\cdots+n_{M-1}2^{\wedge}M_0-1)/2^{\wedge}M_0]$$

$$= Z_n(n_{M-1}n_{M-2}\cdots n_1 n_0)\ \cancel{Z^{(M)}(n_{M-1}\cdots n_1 n_0)}$$

$$\cancel{= \sum_{d_0=0,1} Z^{(M-1)}(d_0 n_{M-2}\cdots n_1 n_0)\bullet [e^{\wedge} j2\pi)^{\wedge} d_0(n_{M_0} + n_{M_0+1}2+\cdots+n_{M-1}2^{\wedge}M_0-1)/2^{\wedge}M_0]}$$

$$\cancel{= \tilde{Z}(n_{M-1}n_{M-2}\cdots n_1 n_0)}$$

Re-ordering pass is added to change $Z_n(n_{M-1}n_{M-2}\bullet\bullet\bullet n_1 n_0)$ in bit reversed order to the normal readout:

$$Z_n(n_0 n_1\bullet\bullet\bullet n_{M-2}n_{M-1}) = Z_n(n)$$

61 PN scrambling $P_R(n)$, $P_I(n)$ = PN code chip n for real and Imaginary axes

Z(n) = PN scrambled Hybrid Walsh encoded chips

$$= Z_n(n)P_2(n)[P_R(n)+j\ P_I(n)]$$

$$= Z_n(n)\mathrm{sign}\{P_2(n)\}\ [\mathrm{sign}\{P_R(n)\} +j\ \mathrm{sign}\{P_I(n)\}]$$ ~~An additional re-ordering pass is added to change the encoded N chip block~~ $\cancel{\tilde{Z}(n_{M-1}n_{M-2}\bullet\bullet\bullet n_1 n_0)}$ ~~in bit reversed ordering to the normal readout ordering~~

$$\cancel{\hat{Z}(n_0 n_1 \bullet\bullet\bullet n_{M-2} n_{M-1}) = \tilde{Z}(n)}$$

~~The fast algorithm in **51** is a computationally efficient means to implement the hybrid complex Walsh encoding of each N chip code block for multiple data rate users whose lowest data rate corresponds to the data symbol rate of an N chip encoded user. The computational complexity of this fast encoding algorithm can be estimated using the computational complexities of the complex Walsh and the DFT fast encoding algorithms, which gives the estimate: $R_A \approx 2M+M_1+2$ real additions per data symbol, and $R_M \approx 2M_1$ real multiplies per data symbol.~~

~~The fast algorithm in **51** consists of M signal processing passes on the stored data symbols, followed by a re-ordering pass for readout of the N chip block of encoded data symbols. Advantage is taken of the equality c=d which allows the d to be used in the code indices for the complex Walsh: $d_m=c_m$, dr=cr, di=ci. Pass 1 implements 2 chip encoding, passes m=2,....,M_0 implement 2^m chip encoding with the complex Walsh codes, passes M_0+1,M_0+2,…,M_0+M_1-1=M-1 implement 2^M_0+m chip encoding with the complex DFT codes, and the last pass M encodes the N=2^M chip data symbols with the DFT codes. This fast algorithm only differs from the fast algorithm in **46** in equations (4) in the use of both the complex Walsh codes and the complex DFT codes with their Kronecker indexing. Unlike the fast algorithm for the real Walsh encoding as well as the algorithm for the complex DFT encoding, the complex Walsh portion of the fast algorithm **51** uses both the sign of the complex Walsh code from the current pass and from the previous pass starting with pass 2.~~

~~The generalization of the fast algorithm in **51** in equations (6) to other Kronecker product constructions for C_N and to the more general constructions for C_N discussed in reference [6] should be apparent to anyone skilled in the CDMA communications art.~~

Equations **(11)** define the fast algorithm for the Rx decoding of the multiple data rate generalized Hybrid Walsh CDMA orthogonal codes for the representative example **58** in equations **(10)**. This fast decoding algorithm implements the decoding of the Rx CDMA encoded chip vector $\hat{Z}(n)$ starting with the removal of the PN scrambling **62** to yield the Rx estimate of the generalized Hybrid Walsh encoded chip vector $\hat{Z}_n(n)$ and followed by an an $M=M_0M_1$-pass computation starting with the M_0-pass computation of the Hybrid Walsh decoding and followed by the M_1-pass computation of the DFT decoding and followed by a re-ordering and rescaling pass to generate the Rx estimate $\hat{Z}(c)$ of the Tx data symbol vector $Z(c)$. Passes $1,2,3,\ldots,M_0$ respectively perform the $2,4,8,\ldots,N_0$ chip Hybrid Walsh decoding and passes $M_0+1,\ldots,M=M_1+M_0$ respectively perform the $2,4,8,\ldots,N_0$ chip DFT decoding. There are other variations to this algorithm such as starting with the computation of c_0 and proceeding to pass M to calculate c_{M-1}.

This fast algorithm is a computationally efficient means to implement the generalized Hybrid Walsh decoding of each N-chip code block for multiple data rate users whose lowest data rate corresponds to the data symbol rate of an N-chip encoded user. It is easily demonstrated that the number of real additions R_A per data symbol is approximately equal to $R_A \approx 2M+M_1+2$ and the number of real multiplies R_M per data symbol is $R_M \approx 2M_1$ in the implementation of this fast algorithm where $N=2^M$. Inclusion of the DFT in the generalized Hybrid Walsh adds some multiplies to the computational complexity with the benefit of increasing the choices for the code length N.

The mathematical definition and the implementation of the fast decoding algorithm for this example of the generalized Hybrid Walsh are sufficiently detailed to enable this algorithm

and implementation to be applied to generalized Hybrid Walsh CDMA codes by someone skilled in the art of CDMA communications and fast transforms.

~~**Receiver equations (7)** describe a representative multiple data rate complex Walsh CDMA decoding for multiple data users for the receiver in FIG. 3 using the definition of the complex Walsh CDMA codes in the invention application [6]. The receiver front end **52** provides estimates~~ $\cancel{\{\hat{Z}(n)\}}$ ~~of the transmitted multiple data rate complex Walsh CDMA encoded chips {Z(n)}. Orthogonality property **53** is expressed as a matrix product of the complex Walsh code chips or equivalently as a matrix product of the complex Walsh code chip numerical signs of the real and imaginary components, for any of the 2,4,8,…,N/2,N chip complex Walsh channelization codes and their repetitions over the N chip code block. The 2-phase PN codes **54** have the useful decoding property that the square of each code chip is unity which is equivalent to observing that the square of each code chip numerical sign is unity.~~

Generalized Hybrid Walsh fast decoding **(11)**
for multiple data rate users
for example **58** in equations **(10)**

62 PN de-scrambling

$\hat{Z}_n(n)$ = PN descrambled CDMA encoded chips $\hat{Z}(n)$

$= \hat{Z}(n)P_2(n)[P_R(n)+j\ P_I(n)]$

$= \hat{Z}(n)\ \mathrm{sign}\{P_2(n)\}[\mathrm{sign}\{P_R(n)\}+j\ \mathrm{sign}\{P_I(n)\}]$

63 Fast decoding of generalized Hybrid Walsh for the example **58** in equations **(10)**

Pass 1 for Hybrid Walsh codes

$Z^{(1)}(c_{M-1}n_1n_2\cdots n_{M-2}\,n_{M-1})$

$$= \sum_{n_0=0,1} \hat{Z}_n(n_0n_1n_2\cdots n_{M-2}n_{M-1}) \bullet [(-1)\hat{}\, n_0\, cr_{M-1} - j(-1)\hat{}\, n_0\, ci_{M-1}]$$

where the Hybrid Walsh indexing reduces to

$$cr_{M_0-1} = cr_{M-1} \bmod(M_1)$$
$$ci_{M_0-1} = ci_{M-1} \bmod(M_1)$$

⋮

~~**Receiver decoding of complex Walsh and hybrid complex Walsh CDMA encoded chips**~~ ~~**(7)**~~

~~52Receiver front end in FIG. 3 provides estimates~~

~~$\{\hat{Z}(n)\}$ **28** of the encoded transmitter chip symbols~~

~~$\{Z(n)\}$ **41** in equations (3)~~

~~53Orthogonality properties of the complex Walsh NxN matrix~~

$$\cancel{\sum_n \widetilde{W}_N(\hat{c},n)\widetilde{W}_N^{*}(n,c) =}$$

$$\cancel{\sum_n [\operatorname{sgn}\{W_N(\hat{cr},n) + j\operatorname{sgn}\{W_N(\hat{ci},n)\}][\operatorname{sgn}\{W_N(n,cr)\} - j\operatorname{sgn}\{W_N(n,ci)\}]}$$

$$\cancel{= 2N\ \delta(\hat{c}, c)}$$

~~where $\hat{c}$, c, n = 0,1,...,N~~

~~$\delta(\hat{c}, c)$ = Delta function of $\hat{c}$ and c~~

~~= 1 for $\hat{c}$ = c~~

~~= 0 otherwise~~

~~cr=cr(c), ci=ci(c) are defined in equations (3)~~

~~54PN de-scrambling of the receiver estimates of the complex and hybrid complex Walsh encoded data chips~~

~~$P_R(n)$, $P_I(n)$ = PN code chip n for real and imaginary axes~~

~~$\tilde{\hat{Z}}(n)$ = PN de-scrambled receiver estimates of the transmitted CDMA encoded chips $\hat{Z}(n)$~~

$$\cancel{= \hat{Z}(n)\,[P_R(n) - j\,P_I(n)]}$$

~~**55a** Complex Walsh decoding and uses a computationally efficient fast encoding algorithm. This algorithm implements the decoding with an M-pass computation. Passes 1,2,3,…,M respectively perform the 2,4,8,…,N chip complex Walsh decoding of the data symbol vector successively starting with the 2 chip decoding in pass 1, the 4 chip decoding in passes 1,2, and the N chip decoding in passes 1,2,3,…,M where $N=2^M$. Using the binary word representations for both d and n, this M pass algorithm is:~~

~~Pass 1:~~

$$\cancel{\hat{Z}^{(1)}(d_{M-1}n_1n_2\cdots n_{M-2}\,n_{M-1}) = \sum_{n_0=0,1} \tilde{\hat{Z}}(n_0n_1n_2\cdots n_{M-2}n_{M-1}) \bullet [(-1)^{\wedge}\, n_0 dr_{M-1} - j(-1)^{\wedge}\, n_0 di_{M-1}]}$$

$\vdots$

Pass m for m=2,…,M_0-1 for Hybrid Walsh codes

$$Z^{(m)}(c_{M-1}c_{M-2}\cdots c_{M-m}n_m\cdots n_{M-2}n_{M-1}) = \sum_{n_{m-1}=0,1} Z^{(m-1)}(c_{M-1}c_{M-2}\cdots c_{M-m+1}n_{m-1}\cdots n_{M-2}n_{M-1}) \cdot [(-1)^{\wedge}n_{m-1}(cr_{M-m}+cr_{M-m+1}) - j(-1)^{\wedge}n_{m-1}(ci_{M-m}+ci_{M-m+1})]$$

where the Hybrid Walsh indexing reduces to

$$cr_{M_0-m} + cr_{M_0-m+1} = [cr_{M-m} + cr_{M-m+1}] \bmod (M_1)$$
$$ci_{M_0-m} + ci_{M_0-m+1} = [ci_{M-m} + ci_{M-m+1}] \bmod (M_1$$

⋮

~~Pass m for m=2,…,M-1~~

$$\cancel{\hat{Z}^{(m)}(d_{M-1}d_{M-2}\cdots d_{M-m}n_m\cdots n_{M-2}n_{M-1})}$$
$$\cancel{=\Sigma\hat{Z}^{(m-1)}(d_{M-1}d_{M-2}\cdots d_{M-m+1}n_{m-1}\cdots n_{M-2}n_{M-1})\cdot}$$
$$\cancel{[(-1)\wedge n_{m-1}(dr_{M-m}+dr_{M-m+1})-j(-1)\wedge n_{m-1}(di_{M-m}+di_{M-m+1})]}$$
$$\cancel{n_{m-1}=0,1}$$

Pass M_0 for Hybrid Walsh codes

$$Z^{(M_0)}(c_{M-1}\cdots c_{M_1}n_{M_0}\cdots n_{M-1})$$
$$=\Sigma\ Z^{(M_0)}(c_{M-1}\cdots c_{M_1+1}\ \ n_{M_0-1}\cdots n_{M-1})$$
$$[(-1)\wedge n_{M_0-1}(cr_{M_1}+cr_{M_1+1})-j(-1)\wedge n_{M_0-1}(ci_{M_1}+ci_{M_1+1})$$
$$n_{M_0-1}=0,1$$

where the Hybrid Walsh indexing reduces to

$$cr_0 + cr_1 = [cr_{M_1} + cr_{M_1+1}] \bmod (M_1)$$
$$ci_9 + ci_1 = [ci_{M_1} + ci_{M_1+1}] \bmod (M)$$

Pass $M_0+m= M_0+1, M_0+2, \ldots, M_0+M_1-1=M-1$ for DFT codes

$$Z^{(M_0+m)}(c_{M-1}\cdots c_{M_1}n_{M_0}\cdots n_{M-m-1}c_{m-1}\cdots c_0)$$

$$= \sum z^{(M_0+m-1)}(c_{M-1}\cdots c_{M_1} n_{M_0}\cdots n_{M-m} c_{m-2}\cdots c_0)$$

$$[e\^{}j2\pi 2\pi n_{M-m}(c_0+c_1 2+\cdots+c_{m-1}2\^{}m-1)/2\^{}m]$$

$$n_{M-m}=0,1$$

Pass M for DFT codes

$$z^{(M)}(c_{M-1}\cdots\ \cdots c_0)$$

$$= \sum z^{(M_0+m-1)}(c_{M-1}\cdots c_{M_1} n_{M_0} c_{M_1-2}\cdots c_0)$$

$$[e\^{}j2\pi\^{}jn_{M_0}(c_0+c_1 2+\cdots+c_{M_1-1}2\^{}M_1-1)/2\^{}M_1]$$

$$n_{M-m}=0,1$$

$$= \hat{Z}(c_{M-1}c_{M-2}\cdots c_1c_0)$$

Reordering and rescaling pass:

$$\hat{z}(c) = \hat{z}(c_0c_1\bullet\bullet\bullet c_{M-2}c_{M-1})$$

$$= (1/4N)\ f[\hat{z}(c_{M-1}c_{M-2}\bullet\bullet\bullet c_1c_0)]$$

$$= (1/4N)\ f[z^{(M)}(c_{M-1}c_{M-2}\bullet\bullet\bullet c_1c_0)]$$

where $f[\hat{z}], f[z^{(M}]$ is the bit reversed value of $\hat{z}$, $z^{(M}$

~~Pass M~~

$$\cancel{\hat{Z}^{(M)}(d_{m-1}d_{m-2}\cdots d_0)}$$

$$\cancel{=\sum\hat{Z}^{(M-1)}(d_{M-1}d_{M-2}\cdots d_1n_0)}$$

~~$[(-1)\hat{}\ n_{M-1}(dr_0+dr_1)\ -j(-1)\hat{}\ n_{M-1}(di_0+di_1)]$~~

~~$n_{M-1}=\ 0,1$~~

~~$=\ N\hat{Z}(d_{m-1}d_{m-2}\cdots d_0)$~~

~~An additional re-ordering pass is added to change the decoded N chip block $\hat{Z}(d_{m-1}d_{m-2}\cdots d_0)$ in bit-reversed ordering to the normal readout ordering. In this representative fast implementation the scaling factor N has been removed in the re-ordering pass whereas a typical implementation will re-scale each pass. The output of this final pass is the receiver estimate of the transmitted data symbol vector~~

~~$\hat{Z}(d_0d_2\cdots d_{M-1}) = \hat{Z}(d)$~~

~~**55b** Hybrid complex Walsh decoding uses a computationally efficient fast encoding algorithm. Similar to the complex Walsh this algorithm implements the decoding with an M pass computation 1,2,3,…,M:~~

~~Pass m= 1,…,M_1 for complex DFT codes~~

$$\cancel{\hat{Z}^{(m)}(n_0\cdots n_{M-m-1}d_{m-1}\cdots d_0)}$$

$$\cancel{= \sum_{n_{M-m}=0,1} \hat{Z}^{(m-1)}(n_0\cdots n_{M-m}d_{m-2}\cdots d_0)\bullet}$$

$$\cancel{[e\hat{}\ j2\pi)\hat{}\ n_{M-m}(d_0+d_1 2+\cdots+d_{m-1}2\hat{}\ m-1)/2\hat{}\ m]}$$

⋮

~~Pass M_1+m=M_1+1,M_1+2,….,M-1 for complex Walsh codes~~

$$-\hat{Z}^{(M_1+m)}(d_{M-1}\cdots d_{M-m}\,n_m\cdots n_{M_0-1}\,d_{M_1-1}\cdots d_0)$$

$$=\sum \hat{Z}^{(M_1+m-1)}(d_{M-1}\cdots d_{M-m-1}\,n_{m-1}\cdots n_{M_0-1}\,d_{M_1-1}\cdots d_0)\bullet$$

$$[(-1)\wedge n_{m-1}(dr_{M_0-m}+dr_{M_0-m+1})-$$

$$-j(-1)\wedge n_{m-1}(di_{M_0-m}+di_{M_0-m+1})$$

$$n_{m-1}=0,1$$

$$\vdots$$

Pass M for complex Walsh codes

$$-\hat{Z}^{(M)}(d_{M-1}\cdots d_1 d_0)$$

$$=\sum \hat{Z}^{(M-1)}(d_{M-1}\cdots d_{M-m-1}\,n_{M_0-1}\,d_{M_1-1}\cdots d_0)\bullet$$

$$[(-1)\wedge n_{M_0-1}(dr_0+dr_1)-$$

$$-j(-1)\wedge n_{M_0-1}(di_0+di_1)$$

$$n_{M_0-1}=0,1$$

$$=\hat{Z}(d_{M-1}d_{M-2}\cdots d_1 d_0)$$

~~An additional re-ordering pass is added to change the decoded N chip block $\hat{Z}(d_{m-1}d_{m-2}\cdots d_0)$ in bit reversed ordering to the normal readout ordering. In this representative fast implementation the scaling factor N has been removed in the re-ordering pass whereas a typical implementation will re-scale each pass. The output of this final pass is the receiver estimate of the transmitted data symbol vector~~

$$\cancel{\hat{Z}(d_0 d_2 \cdots d_{M-1}) = \hat{Z}(d)}$$

~~The fast decoding algorithms **55a, 55b** perform the inverse of the signal processing for the encoding **41, 51** in equations (3),(6) of the complex, hybrid complex Walsh respectively, to recover estimates $\{\hat{Z}(d)\}$ of the transmitter user data symbols $\{Z(d)\}$. These algorithms are computationally efficient means to implement the complex and hybrid complex Walsh decoding of each N chip code block for multiple data rate users whose lowest data rate corresponds to the data symbol rate of an N chip encoded user. For the fast complex Walsh decoding algorithm in **55a** the number of required real additions R_A per data symbol is approximately equal to RA≈2M+2 which is identical to the complexity metric for the fast encoding algorithm. For the fast hybrid complex Walsh decoding algorithm in **55b** the computational complexity is $R_A \approx 2M+M_1+2$ real additions per data symbol and $R_M \approx 2M_1$ real multiplies per data symbol which is identical to the complexity metric for the fast encoding algorithm.~~

~~For the complex Walsh decoding the fast algorithm **55a** implements M signal processing passes on the N chip block of received data chips after de-scrambling, followed by a re-ordering pass of the receiver recovered estimates of the data symbols. Passes m=1,2,….,M implement 2^m chip decoding. For the hybrid complex Walsh the fast algorithm **55b** combines the complex~~

~~Walsh algorithm with a DFT algorithm in M signal processing passes where $M=M_0+M_1$ with M_0, M_1 respectively designating the complex Walsh, DFT decoding passes. Passes $m=1,\ldots,M_1$ implement the complex DFT decoding and the remaining passes $M_1+1,\ldots.,\ M-1$ implement decoding with the complex Walsh codes, and the last pass M completes the complex decoding.~~

FIG. **5A** depicts a representative implementation block diagram for the Tx fast encoder algorithm in example **58** in equations **(10)** for multiple data rate generalized Hybrid Walsh CDMA encoding and replaces the real Walsh encoding **13** in FIG. **1A.** Received data symbols **434** are mapped by the Mux algorithm **436** into the data symbol vector Z(c) memory Mem **435.** The data symbol vector Z(c) is encoded with an $M=M_0M_1$-pass computation starting with the M_0-pass computation of the Hybrid Walsh encoding **437** and followed by the M_1-pass computation of the DFT encoding **438** to yield $Z^{(M)}$ which is reordered in another pass and handed over to the encoded vector $Z_n(n)$ memory Mem **439.** This vector **440** is scrambled by the long and short PN codes **441** to generate the CDMA encoded chip vector Z(n) **442.**

~~**FIG. 5 complex/hybrid complex Walsh CDMA encoding** is a representative implementation of the complex and hybrid complex (complex/hybrid complex) Walsh CDMA encoding which replaces the current real Walsh encoding **13** in FIG. 1, and is defined in equations (3) and (6). The input user data symbols $\{Z(u_{m,k_m})\}$ **56** are mapped into the data symbol vector **57** Z(d) as described in equations (3). Data symbols {Z(d)} are encoded and summed over the user data symbols in **58** and **59** by the fast encoding algorithm in equations **41** in (3) for the complex Walsh and in equations **51** in (6) for the hybrid complex Walsh. For the hybrid complex Walsh, the fast complex DFT encoding **59** follows the fast complex Walsh encoding **58.** This encoding and summing over the user data symbols is followed by PN encoding with the~~

~~scrambling sequence $[P_R(n)+jP_I(n)]$ **60**. Output is the stream of complex CDMA encoded chips $\{Z(n)\}$ **61**.~~

~~It should be obvious to anyone skilled in the communications art that this example implementation in FIG. 5 clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.~~

FIG. 6A depicts a representative implementation block diagram for the Rx fast decoder algorithm in equations (11) for the example 58 in equations (10) of the multiple data rate generalized Hybrid Walsh CDMA decoding and replaces the real Walsh decoding 27 in FIG. 3A. Inputs 443 are the Rx estimates $\hat{Z}(n)$ of the Tx CDMA encoded chip vectors Z(n). Long and short PN scrambling codes is are removed 444 from $\hat{Z}(n)$ to yield the Rx estimate $\hat{Z}_n(n)$ 445 of the Tx Hybrid Walsh encoded chips $Z_n(n)$. The $Z_n(n)$ is decoded by the generalized Hybrid Walsh fast decoding algorithm in equations (11) by executing an $M=M_0M_1$-pass computation starting with the M_0-pass computation of the Hybrid Walsh decoding 446 and followed by the M_1-pass computation of the DFT decoding 447 to yield $Z^{(M)}$ which is reordered and rescaled by multiplying by the factor (1/4N) and handed off to the $\hat{Z}(c)$ memory Mem 448 for de-multiplexing (De-Mux) 449 to yield the Rx decoded estimates $\{\hat{Z}(u_{m,k_m})\}$ 450 of the Tx data symbols $\{Z(u_{m,k_m})\}$ 434 in FIG. 5A.

~~**FIG. 6 complex/hybrid complex Walsh CDMA decoding**~~ ~~is a representative implementation of complex/hybrid Walsh CDMA decoding which replaces the current real Walsh decoding **27** in FIG. 3 and is defined in equations (7). Inputs are the received estimates of the complex CDMA encoded chips $\{\hat{Z}(n)\}$ **62**. The PN scrambling code is stripped off from these chips **63** by~~

~~changing the sign of each chip according to the numerical sign of the real and imaginary components of the complex conjugate of the PN code as per the decoding algorithms **54** in equations (7).The complex/hybrid complex Walsh channelization coding is removed by the fast decoding algorithms in equations **55** in (7) for the complex/hybrid complex Walsh, to recover the receiver estimates~~ $\cancel{\{\hat{Z}(d)\}}$ ~~of the transmitted data symbols {Z(d)}. The complex Walsh fast decoding **64** is followed by the complex DFT fast decoding **65** for the hybrid complex Walsh. Decoded outputs are the estimated data vector~~ $\cancel{\hat{Z}(d)}$ ~~**66** whose entries are read out as the set of receiver estimates~~ $\cancel{\{\hat{Z}(u_{m,k_m})\}}$ ~~**67** of the transmitted data symbols.~~

~~It should be obvious to anyone skilled in the communications art that this example implementation in FIG. 6 clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.~~

~~**Preferred embodiments** in the previous description is provided to enable any person skilled in the art to make or use the present invention. The various modifications to these embodiments will be readily apparent to those skilled in the art, and the generic principles defined herein may be applied to other embodiments without the use of the inventive faculty. Thus, the present invention is not intended to be limited to the embodiments shown herein but is not to be accorded the wider scope consistent with the principles and novel features disclosed herein.~~

~~It should be obvious to anyone skilled in the communications art that this example implementation of the complex Walsh and hybrid complex Walsh for multiple data rate~~

~~users in equations (3),…,(7) clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches. For example, the Kronecker matrices E_N and H_N can be replaced by functionals.~~

~~For cellular applications the transmitter description which includes equations (18) describes the transmission signal processing applicable to this invention for both the hub and user terminals, and the receiver corresponding to the decoding of equations (18) describes the corresponding receiving signal processing for the hub and user terminals for applicability to this invention.~~

~~For optical communications applications the the microwave processing at the front end of both the transmitter and the receiver is replaced by the optical processing which performs the complex modulation for the optical laser transmission in the transmitter and which performs the optical laser receiving function of the microwave processing to recover the complex baseband received signal.~~

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[3] IEEE Communications magazine September 1998 Vol. 36 No. 9, "Wideband CDMA"

[4] IEEE Personal Communications April 1998 Vol. 5 No. 2, "Third Generation Mobile Systems in Europe"

[5] K.G. Beauchamp's book "Walsh functions and their Applications", Academic Press 1975

[6] Patent application, filed Jan. 9, 2001, by U.A. von der Embse

DRAWINGS

FIG. 1 CDMA Transmitter Block Diagram

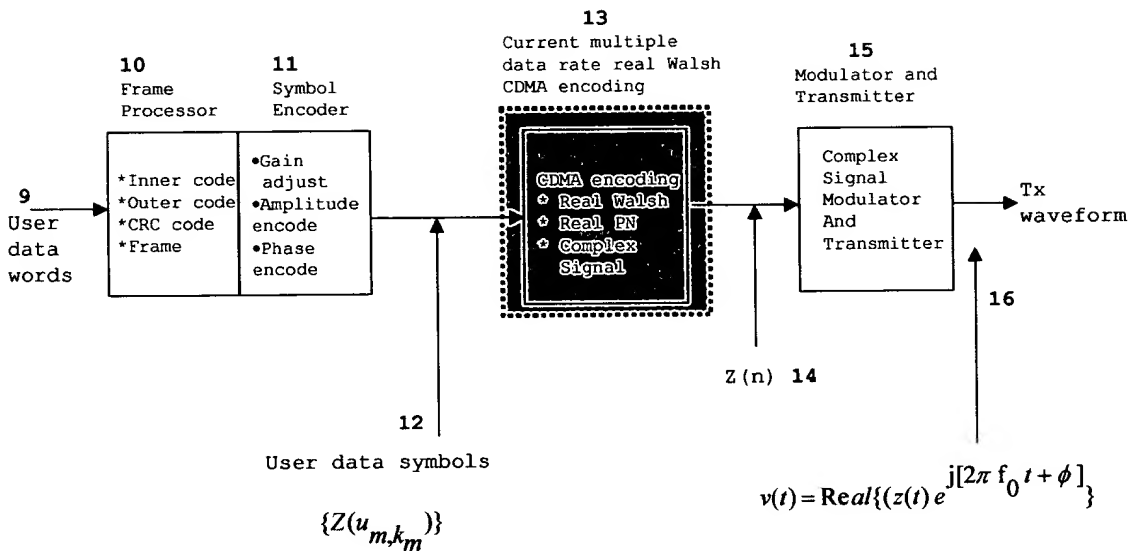


FIG. 2 Multiple Data Rate Real Walsh CDMA Encoding

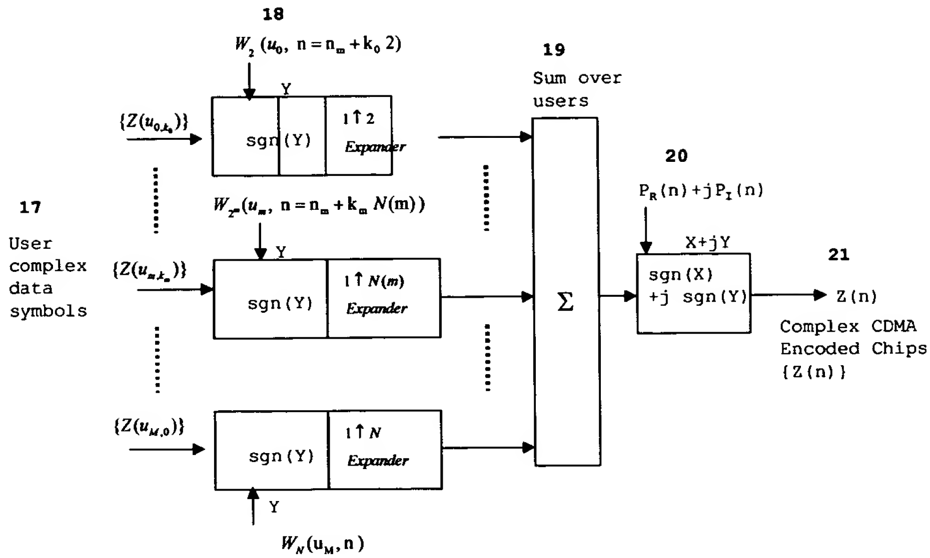
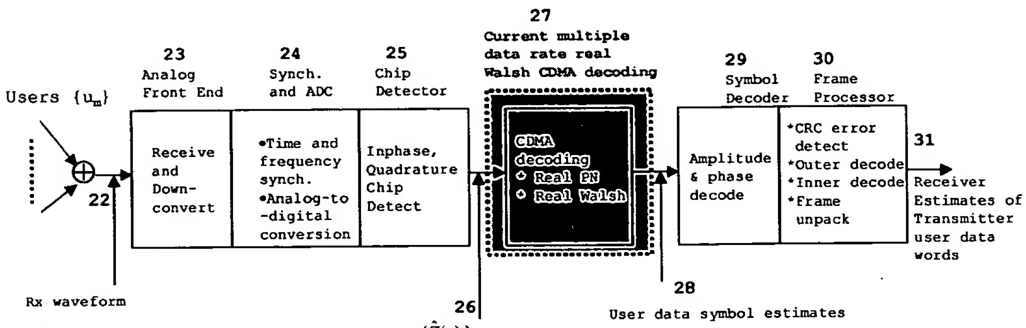


FIG. 3 CDMA Receiver Block Diagram



$\hat{v}(t) = e^{j[2\pi(f_0+\Delta f)t + \Delta\theta]}\}$

$\{\hat{Z}(n)\}$

$\{\hat{Z}(u_{m,k_m})\}$

FIG. 4 Multiple Data Rate Real Walsh CDMA Decoding

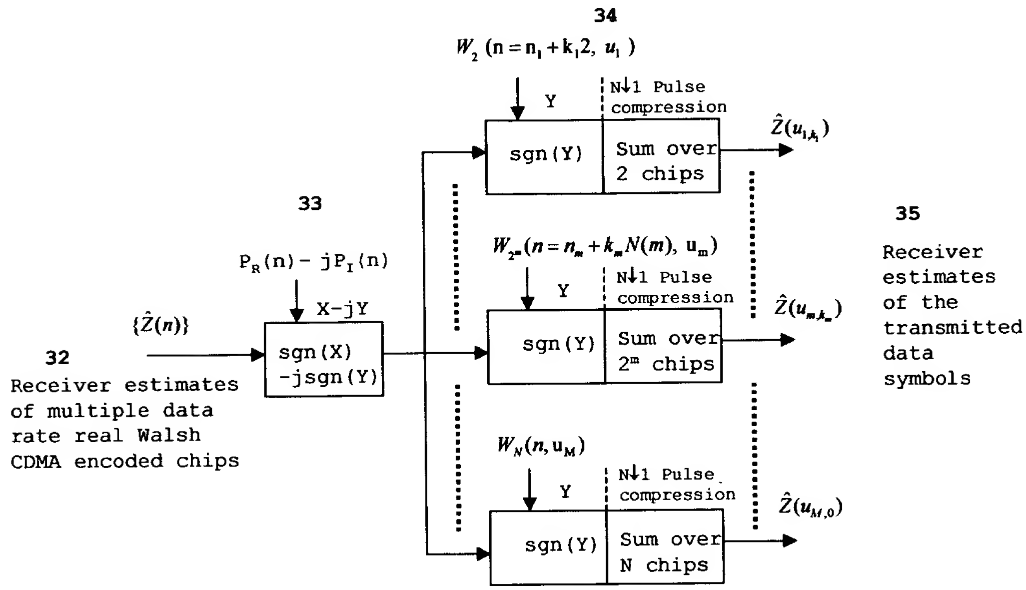


FIG. 5 Complex/Hybrid Complex Walsh CDMA Encoding for Multiple Data Rate Users

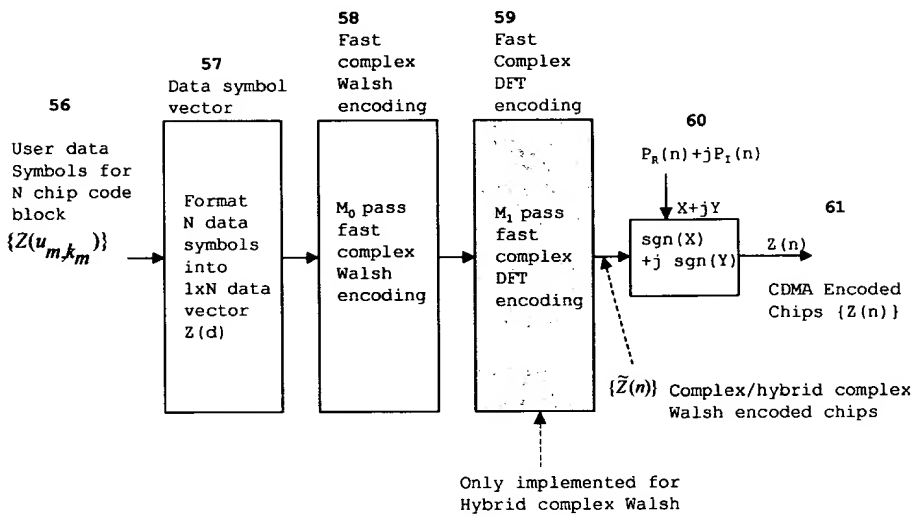
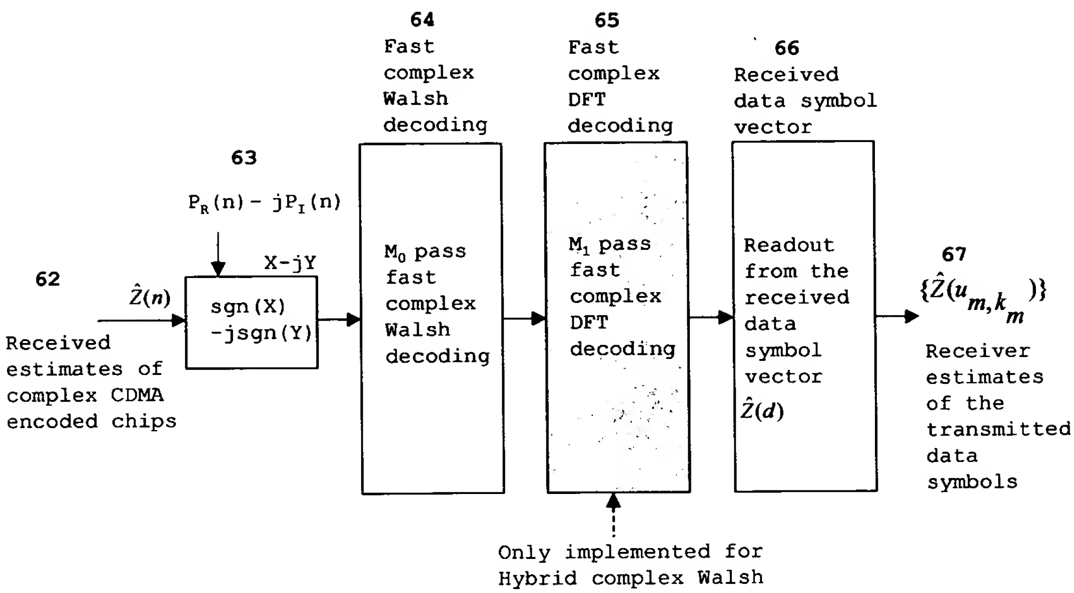


FIG. 6 Complex/Hybrid Complex Walsh CDMA Decoding of Multiple Data Rate Users



FEB 2 2 2005

APPLICATION NO. 09/846.410

TITLE OF INVENTION: Multiple Data Rate ~~Complex~~ Hybrid Walsh Codes for CDMA

INVENTOR~~S~~: Urbain A. von der Embse

FEB 2 2 2005

CLAIMS

WHAT IS CLAIMED IS:

~~1. A means for the implementation of new fast algorithms for complex Walsh orthogonal CDMA encoding and decoding of multiple data rate users over a CDMA frequency band with properties which~~

~~provide a complex Walsh orthogonal code with the real component equal to the real Walsh orthogonal code, and with the imaginary component equal to a reordering of the real Walsh orthogonal code which makes the complex Walsh orthogonal code the correct complex version of the real Walsh orthogonal code to within arbitrary angle rotations and scale factors~~

~~provide complex Walsh orthogonal CDMA codes which reduce to the real Walsh orthogonal CDMA codes upon removal of the imaginary code components~~

~~provide a means to encode and decode multiple data rate users with complex Walsh orthogonal codes for simultaneous transmission over the same CDMA frequency band with computationally efficient algorithm means to implement the encoding and decoding~~

~~provide a computationally efficient algorithm mmeans to encode and decode multiple data rate users with complex Walsh orthogonal codes with values +/-1 +/-j, for simultaneous transmission over the same CDMA frequency band~~

~~2. A means for the implementation of new hybrid complex Walsh orthogonal CDMA encoding and decoding of multiple data rate users over a CDMA frequency band with properties~~

~~provide a means for the construction of hybrid complex Walsh orthogonal CDMA codes which are functional combinations of the complex Walsh, discrete Fourier transform (DFT), Hadamard (real Walsh), and other orthogonal codes and which offer wider choices of code lengths~~

~~provide a means to extend the complex Walsh orthogonal CDMA codes to include the complex discrete Fourier transform (DFT) codes and other orthogonal codes to allow greater flexibility in the choices for the code lengths~~

~~provide new fast algorithm means for the encoding and decoding of hybrid complex Walsh codes for multiple data rate users~~

~~3. A means for the design of hybrid complex Walsh orthogonal CDMA encoding and decoding of multiple data rate users over a CDMA frequency band with properties~~

~~provide a means to provide greater flexibility in the selection of the code length by combining the complex Walsh orthogonal CDMA codes with the complex DFT orthogonal CDMA codes as well as with other orthogonal codes~~

~~provide a Kronecker product means to combine the complex Walsh orthogonal CDMA codes with complex DFT orthogonal CDMA codes as well as with other orthogonal CDMA codes t~~

~~provide a direct sum means to combine the complex Walsh orthogonal CDMA codes with complex DFT orthogonal CDMA codes as well as with other orthogonal CDMA codes~~

~~provide a functionality means to combine the complex Walsh orthogonal CDMA codes with complex DFT orthogonal CDMA codes as well as with other orthogonal CDMA codes~~

~~provide new fast algorithm means for the encoding and decoding of hybrid complex Walsh codes for multiple data rate users~~

~~4. A means to provide unconstrained flexibility in the selection of the code length by functional combining of appropriate orthogonal CDMA codes drawn from a set of code candidates that include the complex Walsh and the complex DFT~~

~~provide a functional means for the generation of orthogonal CDMA codes with unconstrained flexibility in the selection of the code length~~

~~provide a fast algorithm means for the encoding and decoding of CDMA codes designed with a functional means for the generation of orthogonal CDMA codes with unconstrained flexibility in the selection of the code length~~

~~provide a functional means for the generation of orthogonal CDMA codes for multiple data rate users with unconstrained flexibility in the selection of the code length~~

~~provide a fast algorithm means for multiple data rate encoding and decoding of orthogonal CDMA codes which are generated by a functional means for multiple data rate users to provide unconstrained flexibility in the selection of the code length~~

5. A method for the design and implementation of fast encoders and fast decoders for Hybrid Walsh and generalized Hybrid Walsh complex orthogonal CDMA channelization codes for multiple data rate users over a frequency band with properties

Hybrid Walsh inphase (real axis) codes and quadrature (imaginary axis) codes are defined by lexicographic reordering permutations of the Walsh code

Hybrid Walsh codes have a 1-to-1 sequency~frequency correspondence with the DFT codes and have a 1-to-1 even~cosine and odd~sine correspondences with the DFT codes

Hybrid Walsh codes take values {1+j, -1+j, -1-j, 1-j} or equivalently take values {1, j, -1, -j} with a (-45) rotation of axes and a renormalization

generalized Hybrid Walsh codes can be constructed for a wide range of code lengths by combining Hybrid Walsh with DFT (discrete Fourier transform), Hadamard and other orthogonal codes, and quasi-orthogonal PN codes using tensor product, direct product, and functional combining

fast encoding and fast decoding implementation algorithms are defined

algorithms are defined to map multiple data rate user data symbols onto the code input data symbol vector for fast encoding and the inverses of these algorithms are defined for recovery of the data symbols with fast decoding

encoders perform complex multiply encoding of complex data to replace the current Walsh real multiply encoding of inphase and quadrature data

decoders perform complex conjugate transpose multiply decoding of complex data to replace the current Walsh real multiply decoding of inphase and quadrature data

6. A method for the design and implementation of encoders and decoders for complex orthogonal CDMA and generalized complex orthogonal CDMA channelization codes for multiple data rate users over a frequency band with properties

complex codes inphase (real axis) codes and quadrature (imaginary axis) codes are defined by reordering permutations of the real Walsh codes

generalized complex codes can be constructed for a wide range of code lengths by combining the complex codes with DFT (discrete Fourier transform), Hybrid Walsh, Hadamard and other orthogonal codes, and quasi-orthogonal PN codes using tensor product, direct product, and functional combining

fast encoding and fast decoding implementation algorithms are defined

algorithms are defined to map multiple data rate user data symbols onto the code input data symbol vector for fast encoding and the inverses of these algorithms are defined for recovery of the data symbols with fast decoding

encoders perform complex multiply encoding of complex data to replace the current Walsh real multiply encoding of inphase and quadrature data

decoders perform complex conjugate transpose multiply decoding of complex data to replace the current Walsh real multiply decoding of inphase and quadrature data

APPLICATION NO. 09/846,410

TITLE OF INVENTION: Multiple Data Rate ~~Complex~~ Hybrid Walsh Codes for CDMA

INVENTORS: Urbain A. von der Embse

ABSTRACT OF THE DISCLOSURE

The invention provides a method and system for the fast encoding and transmission of simultaneous multiple data rate users for Hybrid Walsh CDMA and generalized Hybrid Walsh CDMA codes over the same frequency band and with the different data rate users separated in the sequency domain of these complex CDMA channelization codes. Sequency separation based on data rate and on quality of service (QoS) is analogous to the corresponding grouping of data rate and QoS users in the frequency domain. There is a 1-to-1 correspondence between sequency for Hybrid Walsh and frequency for DFT (discrete Fourier transform) codes. Fast encoding and decoding algorithms and implementations are presented for the Hybrid Walsh and generalized Hybrid Walsh codes.~~The present invention describes new multiple data rate algorithms for complex Walsh and hybrid complex Walsh orthogonal CDMA channelization encoding and decoding of multiple data rate users, which generate a means to accomodate multiple data rate users over the same CDMA frequency band using complex Walsh and hybrid complex Walsh orthogonal codes. Complex Walsh and hybrid complex Walsh orthogonal CDMA codes have been disclosed in a previous patent application for constant data rate communications. The means of this invention is to provide complex Walsh and hybrid complex Walsh with the means to separate the different data rate users in the sequency domain of the~~

~~complex Walsh analogous to the current use of different frequency bands for the different data rate users. Sequency for complex Walsh and hybrid complex Walsh codes is the average rate of phase angle rotations of the code vectors, and is analogus to frequency in the Fourier domain.~~

~~Current art uses algorithms to generate multiple code length real Walsh CDMA orthogonal codes for the next generation wideband CDMA (W-CDMA), which are orthogonal variable spreading factor (OVSF) CDMA codes. Variable spreading factor refers to a variable code length. The present invention provides a means to significantly improve CDMA performance for multiple data rate users by allowing the use of the new complex Walsh and hybrid complex Walsh CDMA orthogonal codes in place of the real Walsh OVSF CDMA orthogonal codes and with implementation means for fast and computationally efficient encoding and decoding.~~

APPLICATION NO. 09/846,410

TITLE OF INVENTION: Multiple Date Rate ~~Complex~~ Hybrid Walsh Codes for CDMA

INVENTOR~~S~~: Urbain Alfred von der Embse

SEQUENCE LISTING

Not Applicable.

APPLICATION NO. 09/846,410

TITLE OF INVENTION: Multiple Date Rate Hybrid Walsh Codes for CDMA

INVENTOR: Urbain Alfred von der Embse

SEQUENCE LISTING

Not Applicable.